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Stability analysis of large amplitude periodic travelling waves in shallow water

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In recent years, an increasing number of damages due to collision of giant waves with ships or platforms has been reported. A recent review on observations of these giant waves, also called freak waves or rogue waves, and on the various physical mechanisms leading to their formation can be found in [1]. Among the known mechanisms, it is believed that the nonlinear self-focussing mechanism plays an important role in connection with rogue wave formation in sufficiently deep water. In the nonlinear self-focussing mechanism, the focussing of the wave energy results from the evolution of intrinsic instabilities of nonlinear travelling periodic wave trains subject, initially, to slowly modulated periodic (side-band) two-dimensional disturbances. The development of this instability, which is also known as the Benjamin-Feir or modulational instability, is well-known and has been extensively studied as a possible route to freak waves occurrence. However, when the non dimensional water depth kh is less than the critical value khc=1.36 the two-dimensional instabilities of the modulational type disappear. Not only the modulational instabilities become oblique (three-dimensional), but also their growth rate diminishes as the water depth decreases.

In fact, most studies on the stability of periodic gravity waves in shallow water are based on approximate models, which assume weakly nonlinearity as well as the existence of a balance between nonlinear and dispersive effects. Certain stability results obtained with various approximate models have been confirmed in the framework of the fully nonlinear equations [2]. However, these results concern only the well-known modulational instabilities, which are related to second-order quartet wave resonance between the basic wave and the disturbances. Further, waves are usually more nonlinear in finite depth and shallow water than the waves on deep water. This leaves the place to have different type of instability occurring in shallow water when the waves are not of small amplitude.

In this work, we consider the linear stability analysis of finite-amplitude traveling periodic on water of finite depth. This study extends existing results to steeper waves and smaller water depth. This also completes the previous work of Bryant [3, 4] and McLean [2]. Some new types of instability are found for shallow water [5]. When the water depth decreases, higher-order resonances lead to the dominant instabilities. In contrast with the deep water case, we have found that in shallow water the dominant instabilities are usually associated with resonant interactions between five, six, seven and eight waves even for moderate nonlinearity of the basic wave. Moreover, in the shallow water cases kh $<$0.3, we have found that the 1st-order KdV cnoidal approximation is practical to find the locations of the new two-dimensional instabilities in the vector space of the perturbation wavenumber.

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