



## Direct Propagation of Probability Density Functions

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Sustainable decisions in hydrological risk management require detailed information on the probability density function (pdf) of model output. Only then probabilities for the failure of a specific management option or the exceedance of critical thresholds (e.g. of pollutants) can be derived. A new approach of uncertainty propagation in hydrological equations is developed that directly propagates the probability density functions of uncertain model input parameters into the corresponding probability density functions of model output. The basics of the methodology are presented and central applications to different disciplines in hydrology are shown. This work focuses on the following basic hydrological equations: 1) pumping test analysis (THEIS-equation, propagation of uncertainties in recharge and transmissivity), 2) 1-dim groundwater contaminant transport equation (GAUSS-equation, propagation of uncertainties in decay constant and dispersivity), 3) evapotranspiration estimation (PENMAN-MONTEITH-equation, propagation of uncertainty in roughness length). The direct propagation of probability densities is restricted to functions that are monotonic increasing or decreasing or that can be separated in corresponding monotonic branches so that inverse functions can be derived. In case no analytic solutions for inverse functions could be derived, semi-analytical approximations were used.

It is shown that the results of direct probability density function propagation are in perfect agreement with results obtained from corresponding Monte Carlo derived frequency distributions. Direct pdf propagation, however, has the advantage that it yields exact solutions for the resulting hydrological pdfs rather than approximating discontinuous frequency distributions. It is additionally shown that the type of the resulting pdf depends on the specific values (order of magnitude, respectively) of the standard deviation of the input pdf. The dependency of skewness and kurtosis of the propagated pdf on the coefficient of variation of input parameter uncertainty is detected to

be non-monotonic with distinctive maxima.