



On optimal solution error covariances in variational data assimilation

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In variational data assimilation the optimality system includes errors of different nature (background errors, observation errors, model errors). If the errors of the input data are random and normally distributed, then for a linearized problem (tangent linear hypothesis) the covariance operator of the optimal solution errors (analysis error covariance matrix) is given by the inverse of the Hessian of the cost function. This is true in the weakly nonlinear problem when the tangent linear model is a good approximation to the original nonlinear model.

The purpose of our contribution is to derive the formulas for the covariance operator of the optimal solution errors through the Hessian of the nonlinear data assimilation problem without the tangent linear hypothesis. We present the developments of the ideas of [1] for the case of statistical errors. We give the statement of the variational data assimilation problem for a nonlinear evolution model to identify the initial condition. The equation of the error of the optimal solution is derived through the errors of the input data using the Hessian of the misfit functional and the second-order adjoint techniques. We derive the formulas for the covariance operator of the optimal solution errors through the covariance operators of the input errors involving the Hessian of the original nonlinear data assimilation problem. Numerical algorithms are developed to construct the covariance operator of the optimal solution error using the covariance operators of the input errors. The work was supported by the program ECO-NET (EGIDE) and by the Russian Foundation for Basic Research (04-05-64481).

Reference: 1. Le Dimet F.-X., Shutyaev V. On deterministic error analysis in variational data assimilation. *Nonlinear Processes in Geophysics*, 2005, 12, pp. 481-490.