



An application of the Bivariate Generalized Pareto Distribution for determining the probability of low flow extremes

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The most frequently applied method for identifying drought events is a threshold one. Putting the threshold on the given run off magnitude Q_0 two low flow indices can be determined. The first – D denotes deficit volume, second one – T its duration. Random variables D, T are obviously dependent, and in many reported cases their linear correlation coefficient exceeds 90 percent.

Consider two-dimensional random variable (D, T) , and assume that its distribution is Bivariate Generalized Pareto (BGPD) (*Tajvidi 1996*) with the tail distribution

$$\bar{H}(d, t) = 1 - H(d, t) = \left(\frac{\bar{F}^p(d) + k\bar{F}^{\frac{p}{2}}(d)\bar{G}^{\frac{p}{2}}(t) + \bar{G}^p(t)}{\bar{F}^p(0) + k\bar{F}^{\frac{p}{2}}(0)\bar{G}^{\frac{p}{2}}(0) + \bar{G}^p(0)} \right)^{\frac{1}{p}}$$
$$0 < k \leq 2(p - 1), \quad p \geq 2$$

where: \bar{F}, \bar{G} are shifted Univariate Generalized Pareto Tail Distributions (UGPD) for deficit volume and duration. Note, that marginal distribution functions of $H(d, t)$ are also Generalized Pareto.

To fit observed low flows to BGPD two stage procedure is carried out. First, for UPGDs, the standard maximum likelihood method is applied. Next for obtained \bar{F}, \bar{G} tail distributions two parameters p, k are estimated also using the maximum likelihood method. Results of applying above distribution to low flow data from the global data set (*Tallaksen, van Lanen 2004*) are presented. Finally the comparison of BGPD method and *Zelenhasic and Salvai (1987)* model for probability of minimal stream-flow estimation is shown.

References

Tajvidi N. 1996, *Characterisation and Some Statistical Aspects of Univariate and Multivariate Generalized Pareto Distributions.*, PhD Thesis,

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