Geophysical Research Abstracts, Vol. 7, 10268, 2005
SRef-ID: 1607-7962/gra/EGU05-A-10268

Applications of non-linear estimation techniques to seismic imaging

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In this paper an overview is given on the application of directional basis functions, known under the name of curvelet frames, to various aspects of seismic processing and imaging, which involve seismic signal separation (e.g. primary-multiple separation); seismic deconvolution (receiver functions); seismic data continuation and sparseness $\&$ continuity enhanced seismic imaging. Key idea behind the approaches is that the optimal sparseness of curvelet frames for seismic data can be exploited when formulating the solution of above inverse problems in terms of a variational problem of the type

$$\hat{\mathbf{x}} = \arg\min_{x} \frac{1}{2}\|\mathbf{d} - \mathbf{F}\mathbf{x}\|_2^2 + \|\mathbf{x}\|_{w,1}. \tag{1}$$

In this expression, $\mathbf{d}$ is the measured noisy data, $\mathbf{F}$ is either a Frame composition operator (augmented system of several basis functions and frames, e.g. curvelets and discrete cosine transforms) or a Frame operator combined with a forward modelling operator such a convolution or demigration operator and $\mathbf{x}$ the unknown Frame coefficient vector. Contrary to the standard form of Tikhonov-regularization, which typically involves the minimization of a quadratic penalty term on the unknown model, our penalty function is designed to bring out the sparseness in coefficient vector. Sparseness is enhanced by minimizing the weighted ℓ^1-norm $\mathbf{w} \geq 0$.

Depending on how sparse the coefficient vector is, above formalism can be used to (i) separate primary reflections from multiples by setting $\mathbf{F}$ to an augmented system of two curvelet frames with weights $\mathbf{w} = [\mathbf{w}_1\,\mathbf{w}_2]^T$ set according to the predicted primaries and multiples; (ii) deconvolve by setting $\mathbf{F} = \mathcal{F}^T\mathcal{P}^T\mathcal{F}C^T$ with $\mathcal{F}$ the Fourier transform, $\mathcal{P}$ the Fourier transformed convolutions kernels (e.g the P-phases of receiver functions) and $\mathbf{C}^T$ the pseudo-inverse $=$ transpose of the curvelet transform;

(iii) to interpolate missing data by introducing a picking operator in the data mismatch term, i.e. $\|\mathbf{d}-\mathbf{F}\mathbf{x}\|_2^2$ becomes $\|\mathbf{P}\left(\mathbf{d}-\mathbf{F}\mathbf{x}\right)\|_2^2$ in which $\mathbf{P}$ sets the missing data to zero; (iv) to image by setting $\mathbf{F}=\mathbf{K}\mathbf{C}^T$ with $\mathbf{K}$ the scattering/demigration operator.

The success of the above approaches depends largely on the sparseness of the Frame vectors. Since curvelet frames contain directional basis functions that localize in both space $\&$ spatial frequency (angle), remain relatively invariant under scattering operators (nearly diagonalize the wave equation) and obtain near optimal non-linear approximation rates, we are arguably in a good position to solve above seismic inverse problems.