



The optimal Tykhonov-Phillips regularization parameter (α -weighted S-homBLE and via repro-BIQUEUE)

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Numerical tests have documented that the estimate of type BLUE of the parameter vector within a linear *Gauss-Markov model* is *not* robust against *outliers* in the stochastic observation vector. It is for this reason that *we give up* the postulate of unbiasedness, but keeping the set-up of a *linear estimation* of homogeneous type. Ever since *Tykhonov* (1963) and *Phillips* (1962) introduced the *hybrid minimum norm approximation solution* (HAPS) of a *linear improperly posed problem* there has been left the open problem to evaluate the weighting factor α between the least-squares norm and the minimum length norm of the unknown parameters. In most applications of *Tykhonov-Phillips* type of regularization the weighting factor α is determined by simulation studies, such as by means of *L-Curve* (*Hansen* 1992) or the *C_p-Plot* (*Mal-lows* 1973), but according to the literature also optimization techniques have been applied. Here we aim at an objective method to determine the *weighting factor* α within the best linear estimation of type α -weighted S-homBLE (α -weighted *Best homogeneously Linear Estimation*, uniform Tykhonov-Phillips regularization) and developed a method of determining the optimal weighted factor (regularization parameter) α by minimizing the trace of the Mean Square Error matrix MSE (A-optimal design) in the general case. This estimation formula is closed, which provides us not only with the optimal regularization parameter but also with more quicker and more practical solutions than by the simulation methods. Further, it has been shown that the optimal ridge parameter k in *ridge regression* as developed by *Hoerl and Kennard* (1970a, 1970b) and *Hoerl, Kennard and Baldwin* (1975) is just the special case of our general solution by A-optimal design. This result is also compared with another optimal reg-

ularization parameter derived by formulas for the repro-BIQUE (reproducing Best Invariant Quadratic Uniformly Unbiased Estimates) of variance components. In a case study, both model and estimators are tested and analyzed with numerical results.