



## Estimation of high resolution meteorological fields based on geostatistical approaches

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### Abstract:

*The meteorological data is a key input for any hydrological model because it mainly determines the volume of runoff from a catchment.*

*Climatological data are usually measured at point locations, while it is necessary to know these values everywhere; therefore, interpolation is required.*

*The objective of this study is to develop interpolation methods for estimation of high resolution meteorological fields, which are not available everywhere.*

The first focus is to present two methods of interpolation, the **Modified Universal Kriging (MUK)** for interpolation of temperature, so called “**KLAM**” (**K**riging with **L**ocal **A**nisotropy **M**odel) method used for interpolation of precipitation, and the second is to show the results of cross-validation and sensibility-analysis of interpolation.

**Keywords:** *Distributed hydrological modelling, interpolation, variogram model fitting, uncertainty estimation of interpolation, global optimization.*

### Kriging overview

The most promising stochastic tool may be the variogram analysis and Kriging (Cressie, 1985). Kriging is called the best linear unbiased estimator, because it provides linear regression estimate, which is unbiased and has minimum error variance. In general, the basic mathematical models are:

1. No trend in the data: Ordinary kriging (**OK**) is used for spatial prediction:

$$Z(s) = \delta(s) \quad (1)$$

1. Trend in the data: Universal kriging (**UK**) is used supposing that the trend is linear:

$$Z(s) = \alpha_0 + \alpha_1 x(s) + \alpha_2 y(s) + \delta(s) \quad (2)$$

where:  $s$  – is the location,

$Z(s)$  – is a random variable at location  $s$ ,

$\alpha_i$  – are coefficients of the linear trend ( $i=0,1,2$ ),

$x(s), y(s)$  – are the coordinates of the location  $s$ ,

$\delta(s)$  – is random process with existing semi-variogram  $\gamma(h)$ ,

and here  $h$  is a distance vector.

According to the kriging theory (Cressie, 1985), an estimated value  $Z^*(s_0)$  can be expressed as a linear combination of the measured values on “ $n$ ” locations:

$$Z^*(s_0) = \sum_{i=1}^n \lambda_i Z(s_i) \quad (3)$$

where the unknown  $\lambda_i$  ( $i=1,2,\dots,n$ ) are constrained by:  $\sum_{i=1}^n \lambda_i = 1$ , which means that the expected value of the error of estimation ( $E[Z^*(s_0) - Z(s_0)]$ ) is zero. Under second-order stationary, the estimation variance for spatial prediction using weighted average of the neighbouring precipitation data:

$$\sigma_E^2(s_0) = E\left\{[Z^*(s_0) - Z(s_0)]^2\right\} = \sum_{i=1}^n \sum_{j=1}^n \lambda_i \lambda_j \gamma(s_i - s_j) - 2 \sum_{i=1}^n \lambda_i \gamma(s_0 - s_i) \quad (4)$$

Hence, determine the vector of weights  $\lambda$  in equation (4) leads to the following mathematical programming problem:

$$\min_{g(\lambda)=0} \sigma_E^2(x_0) \quad , \text{ where } g(\lambda) := \sum_{i=1}^n \lambda_i - 1 \quad (5)$$

The solution of problem (5) can be found in Cressie (1985).

### **Draft description of interpolation method for mapping temperature data**

In general, ordinary kriging is used for spatial prediction when there is no trend in the data. When there is a trend in the data, we use universal kriging (**UK**). Because the air

temperature is rather correlated with the elevation than with the location of points, it was necessary to modify the basic equation of UK (??) as:

$$T(s) = \beta A(s) + \delta(s) \quad (5)$$

where:  $s$  – is the location,

$T(s)$  – is the estimated temperature,

$A(s)$  – is the altitude at location  $s$ ,

$\beta$  – is the coefficient of  $A(s)$

$\delta(s)$  – is a random process with existing variogram.

In order to use the experimental semi-variogram information in kriging it is necessary to model the experimental semi-variogram (see equation 4). Three theoretical semi-variogram models were tested: exponential, Gaussian, and spherical. Following many several tests, the spherical semi-variogram model was chosen (supposing that no nugget effect, in other words, the measurement error/variability is negligible or equal to zero):

$$\begin{cases} \gamma(h) = c \cdot \left[ 1.5 \frac{h}{a} - 0.5 \left( \frac{h}{a} \right)^3 \right] & h \leq a \\ \gamma(h) = c & h > a \end{cases} \quad (6)$$

where:  $a$  – is the spherical model range.

$c$  – is the spatial variance (sill value when  $h = a$ );

This approach of interpolation can give more fine detailed texture of interpolated temperature than many other methods. In actual practice, uses of this method can be particularly significant in case of estimate accumulation and melting of snow.

The above mentioned interpolation technique was programmed on FORTRAN language with GINO graphics development tool, using OPENGL API, and also was developed a user-friendly graphical program-system for interpolating temperature data in automatic way on large-scale and long term.

### **Draft description of interpolation method for mapping precipitation data**

Interpolation of precipitation is more complicated than for example interpolating temperature or other parameters, because the precipitation is strongly anisotropic process, although sometimes it isn't detectable in a "small" scale. However, real spatial phenomena often show complicated, non-linear spatial structures. Measurements along a particular non-linear spatial path may be highly correlated in dynamic process application, while the other direction shows a little or no correlation. Such processes are called anisotropic.

*The main purposes of our work were to develop an interpolation technique and software which can describe this above mentioned phenomenon.*

The most full of promise tool for studying spatial variability may be the variogram since it can be used in the kriging process to estimate values of the precipitation at an unsampled location in the field. Several researchers have studied the spatial variability of precipitation, for instance M. Amani and T. Lebel (Amani, Lebel, 1997), E. Todini (Todini, 2001), etc. But these – and many other – studies are doesn't touch the topic of “nonlinear anisotropic” whereas it is very important.

*One possible way to describe these complicated directional effects in the correlation is to use sequential local linear coordinates.*

*Summarily, our new model is based on combination of “Ordinary Kriging” (see equation 1) taking spherical experimental variogram.*

*In order to find an optimal parametrization of KLAM model, a general class of globally convergent (derivate-free) univariate optimization algorithm is used.*

The above outlined **KLAM** interpolation technique was programmed on FORTRAN language with GINO graphics development tool, using OpenGL API, and also was developed a user-friendly graphical program-system for interpolating precipitation data in automatic way on large-scale and long term.

### **Cross-validation and sensibility-analysis of interpolation**

*For examination of interpolation efficiency the new algorithm and the well-known Nearest Neighbor and Inverse Distance methods are compared.*

### **Summary**

- A new algorithm for interpolation high resolution precipitation fields has been developed. An approved geostatistic method also was applied in the case of temperature.
- The main advantage of using these methods is that they allow the estimation of uncertainty of interpolated fields.
- It has been proved statistically that applying these methods the uncertainty is consequently smaller comparing to other popular methods.
- Giving possibility to analyze influence of calculated uncertainty on river runoff.

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