



## **Wavelet and Fourier ellipse analysis of a complex-valued time series**

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The analysis of a two-component, or complex-valued, signal (e.g. horizontal velocity) is approached using frequency- and possibly time-dependent ellipses as a basic building block. In both the Fourier and wavelet cases, the ellipse phase, orientation, and eccentricity appear directly as random variables controlling the observed second-order statistics. There are two sensible ways to form a  $2 \times 2$  spectral matrix, and these are independent of variations in the ellipse phase and orientation, respectively. Random eccentricity, meanwhile, plays a fundamentally different role from the other two variables. The relevance of these ideas to multi-taper and multi-wavelet analysis is illustrated with an application to moored current meter records of the internal wave field in the Labrador Sea.

For ellipses with strongly time-varying periods, ridge analysis using the continuous wavelet transform provides a natural framework for estimation of time-varying parameters. Instantaneous frequency curves, and hence ridges, for the two Cartesian components and for the two rotary (analytic and anti-analytic) components are only the same for constant ellipse geometry. More generally, a difference between ridges in the two Cartesian components is equivalent to, and may be used to solve for, time-variable ellipse properties. Period-averaged quantities may then be obtained with the aid of elliptic integrals. This method is illustrated on the problem of inferring the properties of oceanic vortices from position records of subsurface drifters.