



Modeling the statistical dependence in hydrology using “Copulas”

C. De Michele (1), G. Salvadori (2)

(1) DIIAR - Politecnico di Milano (carlo.demichele@polimi.it), (2) Dipartimento di Matematica, Università di Lecce (gianfausto.salvadori@unile.it)

In many hydrological problems several are the (random) variables that play a significant role in the modelization of the phenomenon under investigation, and such variates are generally not independent. For instance, different combinations of rainfall intensity and storm duration may generate storms showing quite different characteristics, and the river management may strongly depend upon the joint features of flood peak and flood volume. Therefore, it is often of fundamental importance to be able to relate the marginal distributions of different variables in order to obtain a joint law describing the main features of the observed hydrological events. Recent advances in applied mathematics have shown that “Copulas” may represent an useful tool for investigating the statistical behaviour of dependent variables. Practically, “Copulas” are operators on the family of (one-dimensional) probability distributions which generate multivariate laws with specified properties. In fact, given two (continuous) random variables X and Y , with marginal distributions F_X and F_Y , there exists a correspondence between their joint law F_{XY} and a proper 2-Copula C , Sklar’s Theorem. The interesting point is that the properties of F_{XY} can be discussed in terms of the structure of C : in fact, it is precisely the copula which captures many of the features of a joint distribution, and dependence properties and measures of association between X and Y can also be investigated in terms of copulas. A further important concept is that of concordance – intuitively, X and Y are concordant if “large” values of one variable tend to be associated with “large” values of the other, and “small” values of one with “small” values of the other; also concordance properties can easily be discussed in terms of 2-Copulas. In this talk, we shall outline the general mathematical framework of copulas, discussing their usefulness in hydrological problems and stressing several possibilities of application.