



Wave capture and wave-vortex duality for internal gravity waves

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New and unexpected results are presented regarding the nonlinear interactions between a small-scale wavepacket and a large-scale vortical mean flow, with an eye towards internal wave dynamics in the atmosphere and oceans and the problem of ‘missing forces’ in atmospheric gravity-wave parametrizations. The present results centre around a wave-breaking scenario termed ‘wave capture’, which differs significantly from the standard wave-breaking scenarios associated with critical layers or mean density decay. We focus on the peculiar wave-mean interactions that accompany wave capture. Examples of these interactions are presented for layerwise two-dimensional, layerwise non-divergent flows in a three-dimensional Boussinesq system, in the strong-stratification limit.

The nature of the interactions can be summarized in the phrase ‘wave-vortex duality’, whose key points are firstly that wavepackets behave in some respects like vortex pairs, as originally shown in the pioneering work of Bretherton (1969), and secondly that a collection of interacting wavepackets and vortices satisfies a conservation theorem for the sum of wave pseudomomentum and vortex impulse, provided that the impulse is defined appropriately. It must be defined as the rotated dipole moment of the Lagrangian-mean potential vorticity (PV). This PV differs crucially from the PV evaluated from the curl of either the Lagrangian-mean or the Eulerian-mean velocity. The results are established here in the strong-stratification limit for rotating (quasi-geostrophic) as well as for non-rotating systems. The concomitant momentum budgets can be expected to be relatively complicated, and to involve far-field recoil effects in the sense discussed in Buhler & McIntyre (2003). The results underline the three-way distinction between impulse, pseudomomentum, and momentum.

While momentum involves the total velocity and pressure fields, impulse and pseudo-momentum involve, in different ways, only the vortical part of the velocity field.