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A inquiry into gravity inversion using a polyhedral model of the Moon

H. Hikida (1), M, Wieczorek (1), and P, Lognonné
(1) Institut de Physique du Globe de Paris

1 A problem of spherical harmonic analysys

The observed gravitational anomalies reflect the mass distribution within the Moon. It is useful to expand the gravitational field in a spherical harmonic form for such a small body. Spherical harmonic analysis, however, has a drawback, if the Bouguer anomalies (the residual anomalies subtracted the topographic contribution from the observed gravitational anomalies) is downward continued within the Moon. Because every coefficient of spherical harmonics of gravity models is affected by the farside gap of the gravity field, spherical harmonic coefficients of the Bouguer anomalies are also affected by the data noise. Moreover, the Bouguer anomalies tend to enhance the signals more as the wavelength of the component of the Bouguer anomalies is smaller, which lead to an unrealistic internal structure model of the Moon.

Werner and Scheeres (1997) derive an analytical expression of gravity fields of a body, using a constant-density polyhedron. Their polyhedral approach remedies the drawback mentioned above; there is a possibility of avoiding the effect of farside gap of the gravity field of the Moon, because the body does not have to be modeled at a uniformly high resolution. Moreover, errors of calculated field can be reduced entirely to errors in the Moon's shape determination and the level of discretization chosen for the shape.

In this study, we construct a suitably discretized polyhedral model of the Moon, and examine the applicability of their polyhedral approach to the gravity inversion as to the crustal thickness of the Moon.

2 Exterior gravitation of a polyhedron

The exterior gravitational potential of a polyhedron is analytically expressed by

$$U = \frac{1}{2} G\rho \left(\sum_{edges} \boldsymbol{r}_e \cdot \boldsymbol{E}_e \cdot \boldsymbol{r}_e \cdot L_e - \sum_{faces} \boldsymbol{r}_f \cdot \boldsymbol{F}_f \cdot \boldsymbol{r}_f \cdot \omega_f \right) \qquad (1)$$

(Werner and Scheeres, 1997), where $\boldsymbol{r}_e$ is a vector from the observational point to the fixed point on the edge of a face, $\boldsymbol{E}_e$ is a dyad in terms of the two face- and edge-normal vectors associated with an edge, L_e is the definite integral in terms of the distances a and b from the field point to the edge's two ends and the edge length e. $\boldsymbol{r}_f$ is a vector from the field point to the arbitrary point on face. $\boldsymbol{F}_f$ is simply the outer product of face-normal vector $\hat{n}_f$.

Eq. (1) shows that we can calculate gravitational potential of a polyhedron by knowing geometrical informations of the polyhedron (*i.e.,* the location of vertices and the direction of a normal vector of each face) as well as the location of the observational point.

3 Model check

To start, we make a preliminary shape model of the Moon by using the Marching Cubes Algorithm (Lorensen and Cline, 1987), which extracts surface information of the model from a 3D field. Because the model's surface is approximated by triangle patches of almost the same size all over the surface, this method may not be suitable to modeling the Moon's surface, avoiding the farside gap of the gravity field.

We calculate gravitational potential of a octahedron, which is enclosed by a sphere of radius R, by using the method of Werner and Scheeres (1997), and compare it with analytical value (*i.e.,* GM/R). The difference of gravitational potential between numerical (*i.e.,* using the method of Werner and Scheeres, 1997) and analytical result at R is nominally 5%. This difference is caused by the fact that the octahedron is not a sphere.

References

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