Geophysical Research Abstracts, Vol. 7, 08480, 2005
SRef-ID: 1607-7962/gra/EGU05-A-08480


# Analysis of kinetic adsorption of solutes in groundwater using Random Walks and the Telegraph equation

**Gerard J.M. Uffink,**

Civil Engineering, Technical University Delft, Delft, The Netherlands

ABSTRACT. Transport of kinetic non-equilibrium adsorbing solutes in groundwater has been investigated. When $N_f$ and $N_a$ (functions of $x$ and $t$) denote concentrations of particles in free (mobile) and adsorbed state, respectively, the model leads to two coupled partial differential equations. In one dimension:

$$\begin{aligned} \partial_t(N_f) + v\partial_x(N_f) &= -\lambda N_f + \mu N_a \\ \partial_t(N_a) &= \lambda N_a - \mu N_a \end{aligned}$$

where $\lambda$ and $\mu$ are state change rates, or transition probabilities per unit of time. A term describing hydrodynamic dispersion, i.e. $D\ \partial^2_{xx}(N_f)$, has been omitted here. In a moving coordinate system where $x$ replaces $x - vt\mu/(\lambda + \mu)$, we can rewrite the system after using $N_f + N_a = u$, $N_f - N_a = w$ and after eliminating $w$. This leads to:

$$(\lambda + \mu)\partial_t u = \lambda\mu \frac{v^2}{(\lambda + \mu)^2}\partial^2_{xx}u - \partial^2_{tt}u - v\frac{\lambda - \mu}{(\lambda + \mu)^2}\partial^2_{xt}u$$

This equation looks like the Telegraph equation. In fact, for $\lambda = \mu$ it equals the Telegraph equation exactly. Similar models appear in the literature in a variety of fields (chemotaxis, heat exchange, chromatography) and a solution is available at least since the 1950's (e.g. Goldstein [2]; Giddings and Eyring [1]). For early times the solution behaves as a wave travelling with a finite speed, while for large times it converges to a Fickian dispersion model with an apparent dispersion coefficient $D^* = \lambda\mu v^2/(\lambda + \mu)^3$. An attractive feature of the telegraph model is that the unrealistic unlimited propagation, so typical of the diffusion model, remains absent. Although for $\lambda \neq \mu$ the equation differs slightly from the classical telegraph equation, the system still behaves essentially in the same way.

The two-dimensional case has been analyzed accordingly and a solution is presented. When, for early times this solution is compared to a 2-D Fickian transport solution some additional anomalies appear. These may very well explain the so-called tailing effect which quite often is observed in field experiments. Accordingly, the paper presents a stochastic procedure (Random Walk) to simulate and visualize the system. Several examples of Random Walk simulations will be presented.

## References


[1] J.C. Giddings and H. Eyring. A molecular dynamic theory of chromotography. *J. Phys. Chem.*, 59:416–421, 1955.

[2] S. Goldstein. On diffusion by discontinuous movements and on the telegraph equation. *Quart. J. Mech. Appl. Math.*, VI:129–156, 1951.