



A formal analysis of the Sandström theorem

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The argument of Sandström or "Sandström theorem" states that the supply and removal of buoyancy at the sea surface do not *drive* the ocean general circulation. The argument - expressed about a century ago - experiences a "renaissance" in recent discussions on the ocean general circulation. Here we present a detailed analysis of the Sandström argument based on the equation for the circulation round a material line. An early formal expression of the argument is shown to be based on three basic assumptions: (i) the flow is strictly steady, so that the material line can be taken to correspond to a streamline and the Coriolis term in the circulation equation trivially vanishes; (ii) the streamline is closed; and (iii) the work done by friction along the line is negative. A new expression is proposed, which only assumes that a statistically steady circulation round a material line exists and that the work done by friction along this line is negative. The recent theorem of Paparella and Young, which is based on the full set of the equations of motion (in the Boussinesq approximation), was claimed to provide a foundation to the Sandström argument. It states that a fluid forced only by a buoyancy contrast at the surface cannot dissipate kinetic energy in the non-diffusive limit. The necessary condition for a minimum of the dissipation integral is that the local friction force identically vanishes (an intuitive result, demonstrable from a variational principle), implying that a steady circulation round any closed contour is precluded in this limit. This clarifies the consistency between the argument of Sandström (a statement about the circulation round a closed contour) and the theorem of Paparella and Young (a statement about the dissipation integral). Both theoretical arguments point to vertical mixing as a driving mechanism of the ocean general circulation.