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Bifurcations sequences in the rotating baroclinic annulus: DNS and laboratory experiments

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1 Introduction

Thermal convection in a fluid subject to differential heating and the influence of strong background rotation occurs in many natural systems, including the atmospheres, oceans and interiors of planets. The ability to model such phenomena with quantitative precision is vital, e.g. for prediction of weather and climate on Earth.

The nonlinear dynamics and routes to chaos of baroclinic waves of a rotating fluid subjected to lateral heating as found in a laboratory experiment, the baroclinic annulus, and in 3D Direct Numerical Simulation (DNS) are discussed here. A fluid in an annular channel, rotating around its vertical axis of symmetry, is subjected to a radial temperature gradient by maintaining the concentric, cylindrical sidewalls at different temperatures. Flows observed in this system cover a vast range of phenomena ranging from a stable baroclinic zonal flow to highly irregular flow, the 'geostrophic turbulence'[1]. For intermediate experimental conditions, a variety of baroclinic waves are observed, which can be steady flow patterns drifting through the annular chamber, or time-dependent with an oscillation of their strength (Amplitude vacillation, AV) or shape (Structural vacillation, SV). More complex flows arise when several wave modes co-exist and undergo mutual nonlinear interactions.

The three main parameters controlling the flow in this system are usually the Prandtl number, $Pr = \nu/\kappa$, the ratio of the kinematic viscosity to the thermal diffusivity, The Taylor number, $\mathcal{T} = 4\Omega^2\nu^{-2}\,(b-a)^5 d^{-1}$, and the stability-parameter or thermal Rossby number, $\Theta = gd\alpha\Delta T\,\Omega^{-2}(b-a)^{-2}$, with a the inner radius, b the outer radius, d the annulus height, Ω the rotation rate of the annulus, ΔT the imposed temperature difference between the side walls, ρ the density, and α the volume expansion coefficient.

The focus of this paper is the transition sequence from a steady wave to chaotic mixed-mode vacillations via amplitude vacillation. The main techniques used are phase space reconstruction, with estimation of dimension and Lyapunov exponents, and a mutual phase coherence measure of the different waves to estimate the strength of the nonlinear triad wave-wave interactions.

2 The experimental results

The apparatus is an annular convection chamber with an inner, cooled side wall of radius 25mm, and outer, heated side of radius 80mm, and a height of 140mm, which is mounted on a turntable[1,2]. The fluid is a water-glycerol mixture. It has long been established that a steady wave may develop amplitude vacillation if the rotation rate or Taylor number, $\mathcal{T}$, is decreased or if the temperature difference or thermal Rossby number, Θ, is increased. This corresponds to the occurrence of amplitude vacillation before a mode transition to the *next-lower* wave number. It has also been found that the parameter range over which a wave shows amplitude vacillation becomes larger as the Prandtl number increases.

Previous experiments[1] on the transition of a steady wave 3 to chaotic modulated amplitude vacillations (3MAV) at a Prandtl number 26 suggested that the 3AV may develop a modulation via a sideband interaction, i.e. a dominant mode 3 interacts with modes 2 and 4. This suggestion was modified later[2], resulting in a more complex interplay of nonlinear wave interactions involving competing wave triads as well the sideband instability and wave-mean flow interactions; steady waves and frequency-locked vacillations (i.e. cases which are more regular than the generic vacillation) show sideband interaction; most quasi-periodic or chaotic flows show mainly resonant triad interaction with the long wave, while irregular flows show little phase coherence.

Experiments at a lower Prandtl number of 13, where the extent in parameter space of the AV and, even more, the MAV regimes was much smaller, have confirmed the general observation from [2]. Some differences were observed, both in terms of the interactions of the same dominant mode in fluids of a different Prandtl number and of flows with different wave numbers at the same Prandtl number.

The main difference between the two values of the Prandtl number was that not only the parametric extent of the vacillating regimes was much smaller in the the lower Prandtl number but also that fewer complex regimes were found between the regular AV and the mode transition to the next lower mode. Furthermore, the degree of phase coherence, indicative of wave-wave interactions was generally much less in the steady flows and the regular AV flows.

3 The numerical results

The mathematical model[3,4] corresponds to the Navier-Stokes equations coupled with the energy equation using the Boussinesq approximation applied to the buoyancy, centrifugal and Coriolis accelerations for air as the working fluid with a Prandtl number of 0.7. The system is made dimensionless by introducing the following reference scales: $\Omega/2$ for time, $g\beta(T_b - T_a)\Omega/2$ for velocity. The scaled temperature is defined as $(T - T_0)/(T_b - T_a)$ with $T_0 = 0.5(T_b + T_a)$, where T_a and T_b are the temperature of the inner and outer radius, respectively. The space variables (r, z) are normalized into the square $[-1, 1] \times [-1, 1]$.

The time integration used is second order accurate and corresponds to a combination of an explicit Adams-Bashforth for the non-linear terms and an implicit backward differentiation formula for the diffusive terms. The space discretization utilizes a pseudospectral collocation Chebyshev polynomials in the meridional plane (r, z) associated with Fourier series in the azimuthal direction. The numerical algorithm is based on an efficient projection scheme to solve the coupling between the velocity and the pressure, which ensures a divergence-free velocity field at each time step. This model has shown its ability to reproduce the expected features over a range of parameter values[4,5].

As one might have expected from the much lower Prandtl number of air, no vacillation was found on increase of Θ or decrease of $\mathcal{T}$. However, time-dependent flows were found at the other end of the parameter range. As the Taylor number, $\mathcal{T}$, is increased, a steady wave of mode 2 first develops a regular amplitude vacillation. This develops later on a quasi-periodic modulation which then becomes chaotic. Figure 3 illustrates the character of the vacillation and its modulation in the return maps of the maximum wave amplitude. The single dot in the first indicates that the vacillation is a periodic 2AV. The closed line in the second indicates that the amplitude maximum in each vacillation cycle follows a regular modulation of a quasi-periodic 2MAV. The third shows that the vacillation is no longer regular. This case is also shown as a time series covering about 14 vacillation cycles. All showed strong phase coherence of mode 2 in the resonant triad with modes 1 and 3, although the phase relationship depended somewhat on the amplitude of mode 2. The degree of coherence did not seem to be affected by the transition to the quasi-periodic 2MAV but was reduced significantly on the transition to the chaotic 2MAV.

4 Conclusions

The laboratory experiment and the DNS showed regime progressions from steady waves to quasi-periodic vacillations and chaotic flows. Nonlinear wave interactions appeared to be strong factors in the transition from regular, steady baroclinic waves to quasi-periodic and chaotic complex mixed-mode flows, where both in the experiment and the DNS a reduction in the phase coherence was associated with a reduction in the 'order' of the flow.

Despite all similarities between experiment and DNS, a major discrepancy is the position of the vacillating states in the parameter space, towards lower $\mathcal{T}$ in the experiment and towards higher $\mathcal{T}$ in the DNS. This apparent discrepancy will be discussed in detail.

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