



## Water transport in unsaturated soils

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### INTRODUCTION

Commonly, the water flow in unsaturated porous media is described by the continuity equation of the volumetric water fluxes and using a modified Darcy's Law. The resulting equation is referred to as Richard's equation, being the governing equation for water movement in unsaturated porous media. This equation is used in most of the numerical codes describing water flow in unsaturated soils.

It is very important to notice that this equation is based on the approach of a representative elementary volume (REV). Usually, it the assumption of a valid REV concept is applied to the quantification of water flow in unsaturated soils, in particular for large scales. However, there exist non-linear soil properties like hysteresis. Detailed numerical investigations of the physical processes at the pore-scale level (e.g. Vogel, 2000) were used to derive the non-linear properties of a REV and to determine the limitations of the application of the Richard's equation.

### THEORETICAL APPROACH

Due to the fact that the Richard's equation is actually a continuity equation, which is by definition also a transport equation, the water flow in unsaturated soils can be formulated as a mass transport problem. The transported mass is the water represented by the water content assuming constant water density, i.e. incompressible conditions. Therefore, the Richard's equation is reformulated using the water content as the independent variable instead of the water tension.

This equation contains a dispersive and an advective term building an advective-dispersive transport equation for the water content. The advective term is determined by the gravity, basically unit gradient flow. The dispersive term is determined by the

capillary forces. The coefficients of both terms are determined by soil specific hydraulic properties, which depend themselves on the water content. It is possible to derive a Peclet number to describe the transport behaviour in dependence of the water content values. As expected, there is a large influence of dispersion at low water content due to the very tortuous shape of the coherent water phase. On the other hand for water content values close to saturation the advective term dominates and the flow is driven mainly by gravity.

## NUMERICAL APPROACH

The detailed structure of the pore space on the microscale and the corresponding spatial variability of the capillary forces are generating the small scale variabilities of the physical properties of a REV. Considering the water content as a stochastic variable on that microscopic scale leads to the definition of a Fokker-Planck equation for the water content evolution, being the stochastic representation of the Richard's equation.

One possibility to obtain numerical solutions for Fokker-Planck equations is the application of the random walk approach, where particles are defined representing the transported quantity. These particles are moved independently and the evolution of their distribution represents the evolution of that quantity. For the evolution of the water content in unsaturated soils this approach has been adapted (Buecker-Gittel et. al., 2003). Each individual particles represent a specified water volume and is moved in accordance to the advection and dispersion parameters.

In order to obtain the water content distribution and to derive those parameters the model domain has to be discretized by cells, the balancing volumes, which are defining the spatial resolution. An additional resolution is given by the number of particles representing the soil specific saturated water content within the cell volume resulting in a quite large total number of particles. Therefore, an iteration for solving the non-linear problem within single time steps must be avoided. The linearisation of the problem is done by calculating the transport parameters using the water content obtained in the previous time step. Because the random walk approach is completely explicit the particle step in each time step has to be smaller than the size of a cell to fulfill the Courant criteria resulting in quite small time step lengths.

## RESULTS AND DISCUSSION

This approach has been applied to simulate the water content evolution in the case of three-dimensional infiltration from point sources, like pipe leaks, with varying infiltration rates. The defined domain was about one by one meter size horizontally and the infiltration source was located about one meter above the groundwater table. The re-

quired computing time was very long due to large number of about 5 million particles. The results showed quite well the evolution of the water content with its fluctuations due to the probabilistic approach.

The main advantages of this approach are, that it is quite simple and provides the possibility to establish parallel computing approaches due to the independent particle movements. Additionally, the approach is free of numerical dispersion beside the discretization by cells and it represents the stochastic properties of the soils. Furthermore, the calculation of solute transport related to the water transport can be done very easily by defining mass loads to the individual particles.

On the other hand, there are some disadvantages of this approach. The random walk approach is transient every time requiring transient calculations even for steady state conditions. Small time step lengths has to be used to fulfill the Courant criteria. Together with the very large number of particles this results in a very long computing time. Only together with parallelization and combination with solute transport calculations this approach can compete with more effective procedures solving the Richard's equation.

#### REFERENCES

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