



Dynamically multiscale adaptive geophysical fluid dynamics simulation using GASpAR

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Adaptive methods for numerically solving PDEs are now being offered by the numerical-methods and associated communities more frequently, are improving in quality and are being applied to a growing number of science applications. At the same time, turbulent-flow problems continue to be prominent in many geoscience areas e.g., meteorology, oceanography, climatology, ecology and solar-terrestrial interactions. Due to the inherent spatio-temporal complexity of such flows, adaptive methods have not often been applied to them. We present a description and initial results of the Geophysical Astrophysical Spectral-element Adaptive Refinement (GASpAR) code, developed as part of the NCAR Geophysical Turbulence Program Initiative. Like most spectral-element codes, GASpAR combines the efficiency of finite-element methods with the accuracy of spectral methods, and it is designed to be flexible enough for a range of geophysics/astrophysics applications where turbulence or other complex multiscale problems arise.

There are several reasons why further progress in geophysical turbulence computation seems to require high-order adaptive methods. One group of reasons stems from the need for high spatial and temporal resolution. When the Reynolds number Re of a turbulent flow is large, nonlinear interactions dominate, and the effective number of degrees of freedom increases as $Re^{9/4}$. Geophysical flows have Re as large as 10^8 to 10^{12} , and so the ability to simulate and examine the multi-scale behavior of geophysical flows depends critically on adequate resolution and/or parameterization of an even larger number of spatiotemporal scales. Theory demands that computations of turbulent flows capture a clear scale separation between the energy-containing inertial and the dissipative scale ranges. Convergence studies on compressible 3D flow computation show that to achieve the desired scale separation between the energy-containing

modes and the dissipation regime using uniform-resolution grids, it is necessary to use at least 2048^3 cells (Sytine et al. 2000). Today such computations can barely be accomplished; a pseudospectral Navier-Stokes code on a grid of 4096^3 regularly spaced points has been run on the Earth Simulator (Isihara et al. 2003) but their Re is still at most of the order of 10^6 , still very far from what geophysics requires.

Another reason for high-order adaptivity is that it is not yet known what *flow structures* are key to understanding the remarkable statistical properties of turbulence (e.g., vortex sheets, spirals or filaments, shocks or fronts, plumes, knots, helices or tubes). The link between structures and non-Gaussian statistics is the basis for the notion of intermittency, which plays a role in e.g., reactive flows, convective plumes, and solar coronal heating. Using traditional techniques, adequate resolution of structures requires extreme computational effort. These considerations suggest that computational adaptivity is needed, provided the dynamically significant structures of the flow are sufficiently sparse that their dynamics can be followed accurately, although embedded in a noisy background.

We have built an object-oriented code, GASpAR, that is flexible enough to be applicable to a wide class of turbulent-flow and other multi-scale PDEs. The computational core is based on spectral-element operators, which are represented as objects. The PDEs are “weakly” formulated as volume integrals with piecewise polynomial basis-function factors $\Phi_I(\vec{x})$ that are each continuous across the global domain, and that interpolate from global node values \vec{X}_I . The solution function is found in $\text{span}_I \Phi_I$, by projecting there from the span of piecewise polynomial basis functions $\phi_j(\vec{x})$ that interpolate from local node values \vec{x}_j but need not be globally continuous. The matrix $\Phi_I(\vec{x}_j)$ generalizes the Boolean “scatter matrix” used in the conforming-element formulation; thus this formulation accommodates both conforming and nonconforming elements, and the implementation includes data structures for handling inter-element communications across parallel processors. Nonconforming h -type dynamic adaptive mesh refinement is provided, and its suitability for turbulent flow models will be examined.