



## Comparison of goodness-of-fit tests adapted to copulas

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In many applied statistical fields, such as hydrology, analyses of multivariate events are of particular interest. For example, design of hydropower dam requires the evaluation of the risk associated with flood peaks at the future dam site. However, it is also important to notice that flood event is best described by three characteristics: flood peak, volume and duration. Considering the correlation between these three features, univariate analysis provides a limited assessment of flood events. Thus, we must use multivariate distribution to estimate the severity of the event of interest. Classical multivariate distributions have been widely used (multivariate normal, bivariate gamma, bivariate Pareto, etc). The drawbacks of those distributions are that the same family is needed for each marginal distribution, and there is not enough choice among those multivariate distributions.

To tackle this problem, copulas reveal to be a very powerful tool. They allow modeling the dependence structure, independently of the marginal functions. A copula is a multivariate cumulative distribution function whose marginals are uniform on the interval (0,1). The link between the marginal functions and the multivariate distribution function is formalized by the copula. Choosing the right family of copula constitutes the main difficulty. Simulations and datasets containing volumes and flood peaks coming from 33 watersheds situated in Province of Quebec allowed us to review the existing procedures.

A preliminary study has been realized, which has permitted to exhibit the degree and the global form of the dependence between volume and flood peaks. Several diagrams have been used (traditional scatterplots, Chi-plot and K-plot), in addition to parametric and nonparametric dependence measures. After this first phase, the method of *Genest*

and Rivest [1993], an empirical method developed to identify the best copula in the Archimedean case was applied. The class of Archimedean copulas is interesting since they can be simulated easily and the computation of measures of dependence is simplified, which allows a straightforward estimation of the parameters. They also present a great diversity in the panel of dependence they can describe. Frank, Clayton and Gumbel families have been used to model the bivariate distribution of volumes and flood peaks on a subset of two rivers. Simulations allowed the comparison of different models, illustrating the crucial choice of the copula family. Goodness-of-fit tests (GOF tests) also represent an approach for checking whether the dependence structure is appropriately modelled by a given family of copulas. The GOF tests recently developed has been applied on real datasets, and evaluated through simulations.

In the Archimedean case, the method developed by *Genest and Rivest* [1993] represents a powerful tool. Among all the Archimedean copulas, some have been accepted to model the bivariate distribution of volumes and flood peaks, some others have been rejected. Concerning the GOF tests, we use simulations to check their power. Three GOF tests, respectively developed by *Chen et al.* [2003], *Fermanian* [2003] and *Genest and al.* [2004] have been implemented. We concluded that the GOF test of *Genest and al.* [2004] is the most adapted to hydrological applications. In comparison, the Fermanian's test appears to be computationally intensive and complex.

#### References:

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