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Binary decomposition analysis (BDA) of a time series

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Hierarchical organization and self-similarity are ubiquitous in natural processes. This explains the power of self-affine and hierarchical methods in geostatistics, with wavelet decomposition being probably the most popular. Still, wavelets, like plain Fourier decomposition or other methods of time series analysis, essentially project a time series $X(t)$ on a set of basis functions $\{E_k(t;n_k)$: k = 0, …, $N\}$: each basis function $E_k(t)$ is multiplied by a constant coefficient a_k and these products $a_kE_k(t)$ are summed to yield the series. As the number of zeroes n_k of the basis functions becomes large, the corresponding coefficients only become small in the case of "well-behaved" processes that generate the time series $X(t)$.

The power and relevance of the concepts and tools provided by the theory of hierarchical scaling seems to offer an alternative avenue to the decomposition, representation, and analysis of time series. This work introduces a statistical method that represents a time series $X(t)$ as a hierarchical tree T_X via its point-wise binary decomposition and uses this tree to study various properties of the series. In our approach, the role of coefficients and basis functions is interchanged: the series is "chopped up" by size, along the ordinate, rather than the abscissa. As a result, the "basis functions" 2^{-k} necessarily become small, whether $X(t)$ is well-behaved or not, while the number of zeroes n_k of the "coefficients" $a_k(t)$ still increases with k.

The mapping of scalar time series into a space of hierarchical trees allows us to switch between temporal and hierarchical ("scaling") domains of quantitative analysis. In our approach, this switching is the counterpart of the duality between temporal and frequency domains in Fourier analysis or in other classical decompositions. We describe

and analyze the topological and metric structure of T_X and show how these structures are connected to the self-affine structure of $X(t)$. We also discuss the correlation analysis of time series in this context. Various applications and properties of BDA are demonstrated using fractional Brownian motions and several observation-derived geophysical time series.