



A stratified, multifractal turbulence/wave model of atmospheric turbulence

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Although there is no argument that the atmosphere is ultimately buoyancy driven via solar radiation, the main turbulent theories of atmospheric dynamics (with various 2-D and 3-D isotropic energy and/or enstrophy cascade regimes) are based on the energy flux ε ; the buoyancy force only plays a secondary role. However, as pointed out by [.....1] and [.....2], the buoyancy force introduces a second quadratic invariant (the flux ϕ_p in addition to the energy flux ε) and this flux ought to be fundamental. In contrast to dynamically based theories, the original Bolgiano-Obukhov theory assumes that buoyancy dominates both the horizontal and the vertical in an isotropic $k^{-11/5}$ "buoyancy subrange": since such a range has never been observed in the horizontal this is too central a role. Indeed, the horizontal wind closely follows the Kolmogorov $k^{-5/3}$ spectrum at least to several hundred kilometers.

We argue in favour of the 23/9D "unified scaling model" [.....3] which for the horizontal wind has a $k^{-11/5}$ energy spectrum in the vertical determined by the buoyancy force variance flux and a $k^{-5/3}$ spectrum in the horizontal determined by the energy flux. We review the empirical evidence of both vertical and horizontal statistics of the horizontal wind finding that the balance of evidence is in favour of the 23/9D model. This is especially true when one recognizes that the 23/9D turbulence can lead to long range biases in the aircraft trajectories and hence statistics and one reinterprets the main horizontal campaigns (GASP, MOZAIC) in terms of the model [.....4]. The problems inherent in *in situ* or aircraft measurements are avoided by the analysis of satellite cloud radiances spanning the scales of 1km to planetary scales [.....5] and indicated

the existence of single scaling regime in the horizontal right through the mesoscale (presumably corresponding to a $k^{-5/3}$ spectrum in the horizontal) whose statistics were very close to those predicted for a multifractal cascade process starting at around 20,000km.

Aside from the 23/9D theory, only the Lumley-Shur [...67] type gravity wave spectral leakage theories simultaneously use both fluxes and can lead to anisotropic scaling ($k^{-5/3}$ in the horizontal, k^{-3} in the vertical, i.e. characterized by $D=7/3$). Although they are unsatisfactory since the driving buoyancy force only acts as a small perturbation to a standard energy flux cascade they have the attraction that they can potentially account for gravity wave phenomena; we return to this question below.

This model assumes that the buoyancy force variance flux $\phi = \Delta f^2/\tau$ (units distance²/time⁵) (where $\Delta f=g\Delta\log\theta$, is the buoyancy force gradient across a layer thickness Δz , τ is the time scale of the transfer) is dominant in the horizontal whereas at the same time, the horizontal structure is dominated by the energy flux $\varepsilon = \Delta v^2/\tau$ where $\Delta v(\Delta x)$ is a horizontal shear in the horizontal wind, and the time scale $\tau = \Delta x/\Delta v$.

The Fractional Integrated Flux model [...8,9] models a velocity field with scaling by fractional integration, i.e. the fourier transform of $v(\mathbf{r})$ is given a power law filter. Note that - as briefly reviewed in [...10] - there is a similar debate about Kolmogorov versus Bolgiano-Obukov scaling in laboratory buoyancy driven turbulence flows (e.g. Benard convection). The 23/9D theory may apply there as well as in the atmosphere.

There is an extensive literature documenting phenomenological evidence for the existence of gravity wave-like undulations in atmospheric structures: we have already mentioned that a turbulent driving mechanism is necessary. The theoretical justification for the gravity wave theory of the spectrum involves postulating a scale separation which would allow one in principle to linearize the atmospheric fields about a large-scale mean flow. These linearized equations are used to determine dispersion relations.

As a first step in reconciling the wave and turbulence approaches we must account for the horizontal / vertical anisotropy. The second step in reconciling the two approaches is to recognize that the 23/9D model is a nonlinear phenomenological model of a highly nonlinear turbulent regime; one does not expect a one-one wavevector-frequency relation. However, it is plausible that it is sufficient to replace the source by the turbulent flux and perform a causal fractional with a dispersion-like kernel. Indeed, it can be shown that in the model, wave groups and energy travel at the usual group velocity. The model can thus be interpreted as the fractional integral of attenuated waves caused by turbulent fluxes. In order to see interesting wave properties in a

cross-section, we must add in the effect of a mean advection velocity which has the effect of adding off-diagonal components to the scale function generator G ; some examples of the standard fractionally integrated flux model and the new turbulence/wave model illustrate the differences. The space-time scaling properties are those predicted by anisotropic turbulence theory discussed above i.e. Kolmogorov statistics in the horizontal and Bolgiano-Obukov statistics in the vertical with multifractal intermittency. Finally, simulations show that as β is decreased, the simulations do indeed become more and more wave-like.

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