



## Pressure dependent permeability models

**F. Zencher**, M. Bonafede

Department of Physics - Section of Geophysics, University of Bologna

e-mail: francesca@ibogeo.df.unibo.it / Phone: +39-051-2095166 / Fax: +39-051-2095058

e-mail: bonafede@ibogfs.df.unibo.it / Phone: +39-051-2095017 / Fax: +39-051-2095058

### Introduction:

Flow through porous media is a topic encountered in many branches of engineering and science. In many applications flow is proportional to the effective pressure gradient according to the Darcy's law:

$$\vec{v}_f = -\frac{K}{\mu} \nabla p$$

where,  $\vec{v}_f$  is the volumetric flow rate per unit area and has the dimensions of velocity,  $\mu$  is the fluid viscosity and  $K$  is the permeability of the porous medium, i.e. a measure of resistance of the porous medium to flow.

A variety of permeability networks have been proposed in the literature, which approximate the structure of different porous materials and provide different expressions for permeability in terms of the network parameters. In particular various authors have observed that the permeability depends on the conditions to which the rocks were subjected and have considered the role of the effective pressure in controlling the evolution of the permeability system within a rock (Bernabe, 1987; Christensen and Ramanantoandro, 1988; Morrow et al. 1986; Olson, 2003). The role of pressure variations accompanying fluid flow on the structure of the permeability network has received minor attention.

Here we want to propose two different models for porous fractured media and to introduce and compare two expressions for a pressure dependent permeability.

### Permeability models:

The first model is well known in literature: the permeability network is represented by a cubic matrix of circular tubes. The expression for the permeability can be found calculating the mean velocity per unit area of the fluid in case of laminar flow, driven by a constant pressure gradient.

If we call the matrix dimension  $b$  and the diameter of the tubes  $\delta$ , the permeability of this system can be written as:

$$K = \frac{\pi}{128} \frac{\delta^4}{b^2}.$$

The dependence of the permeability on the pressure can now be found assuming that the diameter  $\delta$  of the tubes depends on the pressure within, that is calculating the radial displacement of the walls of the tubes in function of the inner pressure  $\Delta p$ .

Calling  $\delta_0$  the diameter of the tubes in absence of inner pressure, the dependence of  $\delta$  on the pressure results:

$$\delta = \delta_0(1 + \Delta p/2G)$$

with  $G$  rigidity of the medium. This implies that we need very high pressure values to make this dependence significant. Being the typical values of the rigidity of the order of  $10^{10}$  Pa, the ratio  $\Delta p/2G$  gives a contribution to permeability negligible in all practical applications.

The second model is instead constituted by layers of intact porous rock with a constant characteristic permeability  $K_r$ , alternated with layers characterized by a periodic distribution of fractures represented by Volterra dislocations.

The effective permeability of the whole system has been calculated and results to have the following expression:

$$K_{eff} = \frac{a_1 \Delta u^3 + a_2 \Delta u + a_3}{b_1 \Delta u^3 + b_2 \Delta u + b_3}$$

where the coefficients  $a_i$  and  $b_i$  are expressions containing the geometrical parameters of the model: the horizontal and vertical distances between the centers of two near dislocations ( $D$  and  $d$ ) and their length  $l$ . The opening  $\Delta u$  of these fractures has been assumed dependent on the pressure within  $\Delta p$ , through the following relation:

$$\Delta u = \Sigma \frac{\pi(1 - \nu)l}{2G} \Delta p$$

with  $\nu$  Poisson's ratio,  $G$  rigidity and  $\Sigma$  a coefficient which represents the interactions among the nearest fractures. For an isolated dislocation  $\Sigma$  is equal to 1. How we can see,  $K_{eff}$  depends on the characteristic permeability  $K_r$ , the inner pressure  $\Delta p$  and the geometrical parameters of the model.

In particular when  $\Delta p$  is equal to 0, all the fractures are closed and the effective permeability is equal to the characteristic permeability  $K_r$ . For relatively high values of the pressure, instead,  $K_{eff}$  tends to an asymptotic value  $K_\infty$  determined by the ratio between  $d$  and  $l$ .

### **Conclusions:**

How we can observe from the expressions for the permeabilities of the two models the dependence on the pressure in the first case is significant only if we consider, for the fluid passing through the tubes of the permeability system, pressure values very high and hardly achievable in crustal rocks.

For the second model instead we can find significative variations for the effective permeability at lower values of pressure and even considering relatively small fracture dimensions relatively low. The asymptotic value  $K_\infty$  determined by the geometrical parameters is easily attained considering pressure values of the order of  $10^6$  Pa.

### **References:**

Bernabe, Y.; 1987: A wide range permeameter for use in rock physics, *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*, 24, 309-315.

Christensen, N. I. and Ramanantoandro, R., 1988: Permeability of the oceanic crust based on experimental studies of basalt permeability at elevated pressures, *Tectonophysics*, 149, 181-186.

Morrow, C.A. and Bo-Chong, Z. and Byerlee, J. D., 1986: Effective pressure law for permeability of Westerly granite under cycling loading, *J. Geophys. Res.*, 91, 3870-3876.

Olson, J. E.; 2003: Sublinear scaling of fracture aperture versus length: An exception or the rule?, *J. Geophys. Res.*, 108 (B9), 2413.