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The need for consistency in spatial discretisation schemes for Richard's equation

B. M. Jackson, H. S. Wheater and A. P. Butler

Department of Civil and Environmental Engineering, Imperial College London, London SW7 2AZ, U. K.

The transport of water through soil, in response to the forces of gravity and water pressure, has been widely represented by the Richards' equation. This parabolic partial differential equation relates the change in soil moisture content to the hydraulic flux divergence. Formally, the first relation employs the law of continuity, while the latter assumes that water flow can be represented by Darcy's law, describing the flow of water in terms of a macroscopic flow velocity vector.

There are only limited special cases where analytic solutions to the equation exist, and therefore most applications require numerical integration. However, the Richards' equation is a particularly difficult one to solve efficiently, as any spatially discretised form of the model is *stiff* (this is a consequence not only of the non-linearity of the soil hydraulic characteristics, but also a result of the parabolic nature of the equation). Furthermore, many authors have noted problems in the simulated fluxes, often specifically related to the averaging scheme used for hydraulic conductivity.

This paper examines common discretisation schemes, and analyses the discretisation errors using Taylor's expansion. It is shown that, provided the time integration algorithm is provided with appropriate error control, the method's *accuracy* is mostly dependant on spatial discretisation, while the *computational cost* relates mostly to the efficiency of the time integration scheme used. For a method to be competitive, there must be some guarantee that the numerical solution will not stray too far from the actual solution path. An important measure of this is whether the problem is satisfied in the limiting case as the stepsize tends to zero. This is the minimum requirement sought in the analysis.

Discretisations of Richards' equation are generally based on one or the other of two

mathematically equivalent forms. In one dimension, these are the local balance formulation;

$$D\frac{\partial \psi}{\partial t} = \frac{\partial \left[K\frac{\partial \omega}{\partial z}\right]}{\partial z} - U \tag{1}$$

or the decomposed formulation.

$$D\frac{\partial \psi}{\partial t} = K\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial K}{\partial z}\left[\frac{\partial \psi}{\partial z} + 1\right]\partial z - U \tag{2}$$

It has generally been reported that schemes based on discretisation of Equation **??** generally perform better than those based on Equation **??**. However, Equation **??** also requires the inclusion of a hydraulic averaging scheme. A variety of conflicting suggestions and studies as to which scheme is best have been made, most of which have been numerically based.

This analysis has addressed the question as to which scheme is better under which condition through an analytical approach. The error terms in the local balance scheme are shown to be generally smaller than in the decomposed scheme, however there are conditions on the local balance scheme to ensure consistency. The arithmetic mean is shown to be the only scheme that will guarantee consistency; in other cases a reduction in order is observed and the scheme fails to converge as the gridlength tends to zero. Restrictions on gridlength changes are also necessary, and in general, the decomposed scheme is recommended for any application requiring changing grids in time. A new discretisation scheme, allowing consistency under less rigourous conditions, is also derived, and presented.