



Polarization of plane waves in viscoelastic anisotropic media

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1 Introduction

Plane waves propagating in viscoelastic anisotropic media are studied as functions of two mutually perpendicular unit vectors \mathbf{n} and \mathbf{m} , and a scalar inhomogeneity parameter $D \in (-\infty, \infty)$. These parameters specify the complex-valued slowness vector \mathbf{p} of a studied plane wave:

$$p_k = \sigma n_k + iDm_k . \quad (1)$$

The vector \mathbf{n} is parallel to the wave propagation direction \mathbf{N} (or it is opposite to \mathbf{N}). The vectors \mathbf{n} and \mathbf{m} specify the propagation attenuation plane, in which the slowness vector (1) is situated. The plane wave is called homogeneous if $D = 0$. In such a case, the propagation and attenuation directions are parallel. The plane wave is called inhomogeneous if $D \neq 0$. In such a case, the attenuation direction differs from the propagation direction but the resulting slowness vector is confined to the propagation-attenuation plane. Thus the inhomogeneity parameter D is a measure of inhomogeneity of the plane wave. The complex-valued parameter σ in (1) must be determined so that the slowness vector (1) satisfies elastodynamic equation.

2 Polarization

Both homogeneous and inhomogeneous plane waves propagating in viscoelastic anisotropic media are, in general, elliptically polarized. Exceptions are the SH plane waves propagating in planes of symmetry of viscoelastic anisotropic media, and P and S waves propagating along some specific directions. The polarization ellipses vary considerably with the direction of propagation \mathbf{N} , with variation of the inhomogeneity parameter D , and with the choice of the propagation attenuation plane. Not only the directions of the axes of the polarization ellipse, but also their eccentricity depends strongly on \mathbf{n} , \mathbf{m} and D . For small values of D and weak attenuation, the polarization is usually nearly linear, i.e., the polarization ellipses are strongly eccentric. With increasing value of inhomogeneity parameter D , the polarization becomes close to circular (small eccentricity). For D large, it becomes more and more difficult (even impossible) to distinguish P and S waves according to their polarization. In such a case, phase velocities of the above waves are getting closer to each other too. Smoothing of anomalous polarization effects known from vicinities of singular directions in perfectly elastic anisotropic media can be observed in vicinities of these directions in viscoelastic anisotropic media.

Two most useful normalization conditions for the polarization vector \mathbf{U} are $U_i \cdot U_i = 1$ and $U_i \cdot U_i^* = 1$. Here U_i are the components of the polarization vector \mathbf{U} , and the superscript $*$ denotes complex conjugacy. The above normalization conditions have no effect on the shape of the polarization ellipses but lead to mutual phase shifts between differently normalized polarizations. The explanation of the phase shift effect is given. The choice of the normalization condition has influence on the solution of the reflection/transmission problem of plane waves at a plane interface separating two viscoelastic anisotropic media. The normalization condition $U_i \cdot U_i = 1$ seems to be more suitable for solution of various wave propagation problems in viscoelastic anisotropic media.

3 Algorithm

For the specification of the slowness vector in the form (1), the so-called mixed specification, the elastodynamic equation for viscoelastic anisotropic media yields a system of linear equations for the components U_i of the polarization vector \mathbf{U} :

$$a_{ijkl}(\sigma n_j + iDm_j)(\sigma n_l + iDm_l)U_k = U_i, \quad i = 1, 2, 3. \quad (2)$$

The polarization vector \mathbf{U} in (2) should satisfy one of the above-mentioned normalization conditions. The complex-valued parameter σ is a solution of the algebraic equation of the sixth degree,

4 Numerical examples

Numerical examples are presented, which exhibit clearly the polarization properties of homogeneous and inhomogeneous plane waves propagating in viscoelastic anisotropic media.