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Weakly two-dimensional interaction of solitons in shallow water

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Summary

We analyse certain geometrical features of interaction of long-crested waves in the framework of two-soliton solution of the Kadomtsev-Petviashvili equation. Such interactions may be responsible for the existence of high-amplitude wave humps. Shown is that the area of extreme elevations is very narrow whereas the extreme slope of the front of the resulting structure may be eight times as high as the maximum slope of the interacting solitons. Interactions of solitons of greatly different heights do not cause extreme elevations but may result in extensive bending of the crests of the counterparts.

Introduction

It has been recently suggested by many authors that an appropriate nonlinear mechanism could be responsible for extreme waves. We concentrate on a specific source for considerable changes in the wave amplitudes, namely, nonlinear superposition of solitary waves in shallow water. A suitable mathematical model for the description of the interaction of soliton-like shallow water waves travelling under slightly different directions is the Kadomtsev-Petviashvili (KP) equation that admits explicit formulae for multi-soliton solutions.

A well-known feature of such interactions is that they may lead to spatially localised extreme surface elevations. For interacting waves with equal amplitudes such high humps resemble Mach stem and can be up to four times as high as the incoming waves. Although known for a long time [1], this mechanism has been only recently proposed as an explanation of the freak wave phenomenon [2,3]. The reason is that this mechanism may become evident only (i) provided long-crested shallow water waves can be associated with solitons and (ii) provided the KP equation is a valid model for such waves. These conditions may be uncommon for storm waves; however, they may be satisfied when two or more systems of swell approaching a certain area from different directions.

Geometry of interactions

The spatial extension of the high hump in the framework of soliton interactions may be associated with the area where the interacting waves have a common crest [2]. We show that for equal amplitude incoming solitons the area of 'truly' nonlinear interactions (where elevation exceeds the sum of amplitudes of the counterparts) may considerably exceed the estimates based on the geometry of the wave crests [4].

Further, we extend a part of the analysis to the case of interacting solitons with unequal amplitudes. The location and the height of the global maximum of the two-soliton solution as well as its symmetry properties are established for the case when the maximum amplitude exceeds the sum of amplitudes of the interacting solitons [5]. The relative increase of the amplitude is largest when the counterparts are equal whereas elevations greatly exceeding the sum of amplitudes of the counterparts only occur when the amplitudes of the intersecting solitons are comparable.

Slope of the interaction pattern

A pronounced feature of freak waves is that they are particularly steep. We show that nonlinear interaction in the framework of the KP equation substantially modifies the profile of the two-soliton solution. The slope of the high wave hump may be 8 times as large as the slope of the equal amplitude incoming waves [4]. This result, although intriguing, is not totally unexpected, because the resonant KdV soliton is higher and therefore narrower than the incoming solitons. For unequal amplitude solitons the amplification of slope of the front of the interaction pattern is shown to be proportional to the amplitude amplification. Thus in this case the nonlinear interaction leads to an extraordinarily high and narrow structure. One could speculate that such wave hump might easily break until it reaches its theoretically maximum height. The possibility of breaking of the high and nonlinear wave hump makes a hit by a near-resonant structure exceptionally dangerous.

Soliton interactions in realistic conditions

Apart from wind-generated rogue waves, the presented mechanism may have an intriguing application in the analysis of abnormally high waves in shallow coastal areas hosting intense high speed ship traffic. The sequences of long-crested soliton-like waves are frequently excited by contemporary ships if they sail at speeds roughly equal with the maximum phase speed of gravity waves [6]. Groups of solitonic waves intersecting at a small angle may appear if wakes from two ships meet each other. Their interaction may be responsible for dangerous waves along shorelines.

Yet the fraction of sea surface occupied by extreme elevations is apparently small as compared with the area of a wave storm, because extreme surface elevation may occur only if the heights of the incoming waves, their intersection angle and the local water depth are specifically balanced.

However, an important difference should be underlined between high waves possibly excited by the described mechanism and those arising owing to focusing of transient and directionally spread waves. In the latter case a number waves with different frequencies and propagation directions are focused at one point at a specific time instant to produce a time-varying transient wave group that normally does not propagate far from the focussing area [3]. A wave hump from nonlinear interaction, theoretically, has unlimited life-time and may cross large sea areas in favourable conditions [3]. Thus, one should account for the expected life-time of nonlinear wave humps (additionally to the sea area covered by extreme elevation at a certain time instant) when estimating the probability of occurrence of abnormally high waves.

Interaction pattern of solitons of greatly different amplitudes

For largely different amplitudes of the interacting solitons, the amplitude amplification remains modest. However, the spatial extent of the influence of nonlinear interaction of solitons with considerably different amplitudes it is roughly as large as if the amplitudes were equal. In this case, the interaction becomes more clearly evident in the form of bending of the crests of both the counterparts. This effect may drastically increase the probability of encountering a hit by a high wave (possibly with a particularly large slope) arriving from an unexpected direction.

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