



local mass conservation and velocity splitting in PV-conserving balanced models

A. R. Mohebalhojeh (1), M. E. McIntyre (2)

(1) Institute of Geophysics, University of Tehran, Iran and School of Mathematics, University of St Andrews, UK, (2) Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK

The most accurate known non-Hamiltonian potential-vorticity-conserving balanced models (PBMs) all fail to conserve mass locally. That is, these PBMs exhibit “velocity splitting” in the sense of having two velocity fields, one to advect and evaluate the exact potential vorticity (PV), and another to advect mass, the difference between the two velocity fields being nonzero in general even if tiny. Unlike the different velocity splitting endemic to Hamiltonian balanced models, in which different velocity fields advect and evaluate the exact PV, the present splitting can be healed. There is an infinite subset of the set of all PBMs that do conserve mass locally and are therefore free of velocity splitting. Such models have the fundamental advantage of possessing a full set of Casimir invariants, including enstrophy and all the higher moments of the isentropic distributions of PV. This subset is characterized and explicitly defined. The best-known member of the subset is the “Bolin–Charney balance model”, or “balance equations”, in the shallow-water and isentropic-coordinate versions. However, there exists within the subset a new class of “hyperbalance equations”, not previously recognized, whose formal accuracies can be made as high as those of any other PBM, though not necessarily their numerical accuracies. Numerical results are presented to examine the accuracy of hyperbalance equations.