



## Asymptotic Single and Multiple Scattering Statistics in Optically Thick Multifractal Clouds

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A current challenge of climate modeling is that general circulation model (GCM) results are extremely sensitive to parameterizations of the poorly understood cloud/radiation interactions. In order to correctly simulate radiation GCMs, we must have realistic models of the cloud statistics over a wide range of scales. Satellite studies have shown that cloud radiances (at both visible and infra red wavelengths) are scale invariant over scales spanning much of the meteorologically significant range, and airborne lidar data have shown that vertical cross sections of passive scalar clouds are also scaling but with quite different exponents in the horizontal and vertical directions (quantified by an “elliptical dimension” =  $D_{el}-23/9$  close to the empirical value  $2.55 \pm 0.02$ ). This multifractality is likely the result of huge cloud variability spanning large scale ranges.

Over the last twenty years, many studies have been devoted to radiative transfer in mono- or multi-scaling cloud fields. Although these studies are of great value, their vast majority have been limited to numerical investigations in relatively optically thin clouds. An exception to this was our earlier development of a formalism where we have presented asymptotic forms for single scattering statistics in optically thick universal multifractal and conservative clouds (i.e. turbulent fluxes, the direct result of turbulent cascades, with nonconservation parameter  $H = 0$ ;  $H = 1/3$  for passive scalar clouds; for aircraft cloud LWC measurements:  $H \approx 0.3$ ). An essential result of this work was that short- and long-photon paths exhibit quite different scaling behaviours: in the near regime the direct transmission is approximately exponential with a renormalized extinction coefficient  $\kappa_{eff} < \kappa$ , where the transmission behaves as if all scattering events were related to the most probable singularity in the cloud density field. In the far regime, the transmission falls off much more slowly (on account of the

so-called “Levy holes”).

In this presentation we show how the latter study 1) can be extended to non-conservative ( $H > 0$ ) general “universal” multifractal clouds, and 2) how the analytic single scattering results can be generalized to multiple scattering - at least for optical thickness below a “super thick” limit ( $\tau \gg 100$ ). Indeed, the theoretical and numerical single scattering analytic results give accurate predictions for the mean cloud optical properties of clouds with realistic multifractal parameters and cloud optical thicknesses. In this contribution we therefore present new results on the transmission statistics in general universal, multifractal clouds (for various  $H$  exponents and levels of intermittency). By “renormalizing” the radiation, we also relate the mean transmission statistics to those of a renormalized homogeneous cloud. To understand the behaviour of photons entering the cloud, recall that for clouds with universal multifractal parameter  $\alpha < 2$  (cloud observations show  $\alpha \approx 1.8$ ) the cloud has both fractal and nonfractal components (the former weak density regions are called “Levy holes”). We find that in the mean over many scatters, the photons move near the edges of the Levy holes - i.e. near the part of the cloud whose density has the highest space filling value (largest  $\gamma$  such that  $c(\gamma) = 0$ ,  $c$  is the codimension of the LWC singularity  $\gamma$ ). This mean behaviour is the result of the photons moving much farther in less dense regions and then getting “stuck” in higher density fractal regions. The result looks like a Levy walk, but it is not; the photon step length variance is finite. These results show that qualitative ideas of photon “channelling” through low density regions is simplistic due to the tendency of photons to intersect dense singularities. By varying the extinction coefficient, we are able to study the effect of increasing cloud thickness, for typical cloud mean optical thickness in the range 10-200. For example, using the observed multifractal cloud characteristics, we predict that the mean cloud transmission decreases with the 0.88 power of the total optical thickness (the corresponding homogeneous exponent being unity). For clouds with a total optical thickness of 100 (with  $1 - g = 0.15$ ) this is a non-negligible 38% effect with respect to homogeneity.

These multiple scattering predictions are numerically tested using the discrete angle radiative transfer (DART) approach in which the radiances decouple into non-interacting families with only four (for 2-D clouds) radiance directions each. Since in thick clouds the phase function is of secondary importance (it doesn’t affect the scaling exponents; the prefactors however do depend on them), this approach is justified because photons actually undergo many scatterings. The steady-state radiative transfer equation is then replaced by a linear algebra problem involving a finite scattering matrix. Using new highly accurate, rapid sparse matrix techniques, one is therefore in a position to solve exactly and rapidly the radiative transfer equation in a great variety of non homogeneous media. Preliminary results concerning the effects of stratification

and more generally of anisotropy (associated with different cloud types) are also presented. Ultimately, these simulations promise to allow us to understand the statistical relation (at all scales) between the radiative fluxes and the cloud liquid water density.