



## Predictability of Lagrangian trajectories

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We address the problem of predicting the position of a current following particle (fluid parcel) or particle cluster in a real flow by using a numerical ocean model. In our preliminary study, both, real and model flows are synthetic with different, but close, parameters (twins experiment). The trajectory of a model particle is viewed as a prediction of a real particle started from the same initial position. The predictability problem splits in two parts. The first task is to estimate the predictability limit (prediction time) (for an individual particle or for the center of particle cluster) given a quantitative discrepancy between real and model parameters. The second part of the problem is to construct an optimal filtering algorithm considering the model trajectories as incomplete noisy observations of unobservable real trajectories. Particularly, we address these problems in the framework of an isotropic stochastic flow which is characterized by three parameters: Lagrangian correlation time  $\tau$ , velocity variance,  $\sigma_u^2$ , and its space correlation radius,  $R$ . In the first part of the talk we investigate dependence of the prediction time on the difference between model and real values of  $\tau$ ,  $\sigma_u^2$ , and  $R$ . In particular, if the model is a smooth version of real, an exact formula for the prediction time can be derived

$$T_{pred} \approx \frac{4}{\Lambda} \ln \frac{R}{h}, \quad (1)$$

where  $\Lambda = \Lambda(\tau, \sigma_u^2, R)$  is the Lyapunov second moment and  $h$  is the smoothing scale. This formula is in good agreement with QG twins experiments. Consequences of (1) for numerical modeling are very strong: significant increasing in the spatial resolution of a model (which is of tremendous computational cost) would produce a small effect on the Lagrangian prediction skill. The reason behind (1) is an exponential growth of the mean square separation for a particle pair in a homogeneous stochastic flow at

intermediate time.

Concerning with optimal filtering, we derive Kushner-Zakai equation for the optimal non-linear prediction (ONP) and then show that the Extended Kalman Filter (EKF) is a zero approximation for the optimal prediction with respect to a small parameter  $\epsilon$  characterizing energy of the unobservable component. Practically, it appears that for fairly small  $\epsilon$  EKF and optimal prediction are indistinguishable. Moreover, it turns out that the traditional center of mass algorithm (ensemble average) is only slightly worse than EKF and ONP while being much more stable and computationally cheaper.

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