



## On the mathematical stability of stratified flow models including a sophisticated turbulence closure scheme

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A large number of present-day models of geophysical and environmental fluid flows rely on Fourier-Fick parameterisations of the vertical turbulent fluxes of momentum, heat, and tracers: the flux of the relevant quantity is expressed as the product of its vertical derivative and a suitably-defined eddy coefficient. Nowadays, the latter is rarely assumed to be a constant. More often than not, it is obtained from sophisticated models. Some of them rely on additional differential equations, which govern the evolution of variables describing the state of turbulent fluctuations.

Occasionally, the numerical simulations based on such models exhibit small-scale oscillations, causing the eddy coefficients to vary over several orders of magnitude. Theoretical developments suggest that these spurious oscillations are due to a lack of stability of turbulence closure schemes.

Let  $\lambda_m$  represent the eddy diffusivity relevant to  $\psi_m$ , the  $m$ -th variable of the model under consideration. By studying the evolution of a small-amplitude and small-time/space scale perturbations a stability condition is obtained: the eigenvalues of the matrix  $\lambda_m \delta_{m,n} + \psi_{m,z} \partial \lambda_m / \partial \psi_{n,z}$  must be positive — where  $\delta_{m,n}$  is the Kronecker symbol and  $\psi_{n,z}$  is the vertical derivative of  $\psi_n$ .

Though strong simplifications are needed to derive the stability criterion above, a series of numerical experiments point to its relevance for a number of water column models relying on turbulence closure schemes ranging from that of Munk-Anderson to the Mellor-Yamada level 2.5 model.

It is believed that the present theory may be applied to all local turbulence closure

schemes applied in atmospheric and oceanic modelling. Whether or not it could be of use for studying the stability of non-local approaches is still an open question.