



A new theory for Rossby waves with faster westward phase speeds

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The original finding of C. G. Rossby's pioneering study in 1939 regarding the westward phase propagation of planetary-scale waves in the atmosphere was developed from the vorticity equation on the unbounded beta-plane. Recent observations made with the altimeter aboard the Topex/Poseidon satellite have repeatedly demonstrated that Rossby's seminal theory predicts phase speeds that are up to a factor of 4 slower than the observed speeds. In the new theory described here the phase speed of planetary waves is calculated in a zonal channel (which in Rossby's original theory merely quantizes the meridional wavenumber) whose width is allowed to vary. Starting from the linearized Shallow Water Equations, assuming that the solutions for (u, v, h) have a zonally propagating wave structure with latitude dependent amplitude and imposing the boundary conditions of no normal (i.e. v) flow through the channel flow results in a nonlinear eigenvalue equation for the wave's phase speed - C . The crux of the new theory is a transformation of the nonlinear eigenvalue equation in C into a linear eigenvalue equation for an eigenvalue, E . the sought phase speed, C , is determined as one of the roots of a cubic equation that relates E to C . The other two roots of this cubic equation are the phase speeds of the Inertia-Gravity (Poincaré) waves. The linear eigenvalue equation for the latitude-dependent wave's amplitude is the Schrödinger equation with quadratic potential and the eigenvalue is simply the allowed energy level.

The primary modification to the classical theory arises from the inclusion of the latitude dependence of the Coriolis frequency so that, unlike Rossby's original theory,

the differential equation has non-constant coefficients. We demonstrate that by eliminating the latitude-dependence of the Coriolis frequency the eigenvalue problem has exact solutions that yield the exact same phase speed as in the classical theory. In the general case, however, the eigenvalues are always smaller than those in for fixed Coriolis frequency so that they always yield larger phase speeds for Rossby waves (and larger phase speeds for Inertia-Gravity waves)! We also show that the difference between the two phase speed depends on the speed of gravity waves – $(gH)^{1/2}$ - and for slow gravity waves (as is the case in equivalent barotropic cases) the difference between the two phase speeds gets quite large.

The analytical results on the beta-plane are extended numerically to spherical coordinates, where the phase speed of Rossby waves gets even larger, reaching four times the speed expected from the classical theory for the same values of the pertinent parameters (i.e. zonal wavenumber, channel width, channel mean latitude and speed of gravity waves). These numerical findings are consistent with the Topex/Poseidon observations.