



Local Nonlinearity and Linearization of On-Off Switches – A Comparison between Two Adjoint Approaches

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Parameterized physical processes often contain discontinuous on-off switches controlled by threshold values of model variables. The perturbed on-off switches are locally nonlinear and their tangent linearization yields delta functions in the perturbation equation. These delta functions generalize the tangent linear operator but can be also treated as "forcing" pulses. In this regard, two adjoint approaches were previously developed for on-off switches: the generalized adjoint by Xu (1996, *J. Atmos. Sci.*, 1123-1155) and the adjoint based on nonlinear perturbation equation (NPE) by Mu and Wang (2003, *J. Atmos. Sci.*, 2010-2018). The two approaches are compared in the tangent-linear limit. The results show that the two approaches are similar in that they both consider the switch time perturbation. As the impact of the switch time perturbation is represented by the delta function, the delta function plays the same role in the two approaches in deriving the costfunction gradient. Both approaches use the classic adjoint for the backward integration over almost the entire time window except for the switch points. The two approaches, however, are different in handling the delta function (at each switch point). In the generalized adjoint, the delta function is carried by the tangent-linear operator and transported to the adjoint, so the additional gradient term caused by the switch time perturbation is automatically considered by the backward integration of the adjoint. In the NPE approach, the delta function is treated as a "forcing" in the perturbation equation and then integrated with the adjoint variable, so the additional gradient term is produced by the "forcing" at the switch time. Thus, although the two approaches lead to the same result, they suggest different computational procedures. These differences are examined in details.