Geophysical Research Abstracts, Vol. 7, 02127, 2005
SRef-ID: 1607-7962/gra/EGU05-A-02127

Morse wavelets, polarization and seismology

S. Olhede (1) and A. Walden (1)
(1) Imperial College London (s.olhede/a.walden@imperial.ac.uk)

A time series is a set of observations of a process where the observations are associated with an ordering. In many areas of statistics a common modelling assumption is that the processes observed over time have reached some equilibrium state. In many instances such an assumption is not reasonable, and so in recent years time-localised analysis has been developed. In general more than a single process would be of interest, as for example the multi-component data of seismic readings of earthquakes.
A common tool in time-localised analysis is the continuous wavelet transform. Using the continuous wavelet transform we can decompose an arbitrary signal $x(t)$ into time-scale contributions. The choice of analysis wavelet ψ is crucial - ideally the wavelet would be well localised in both time and frequency simultaneously, or non-zero for only very limited ranges of times and frequencies, but due to the uncertainty principle this is naturally impossible. To find wavelets with good time-frequency localisation Daubechies and Paul (1988) introduced the concept of a localisation operator $\mathcal{P}_{\mathcal{D}}$, which restricts the signal $x(t)$ to an arbitrary time/scale domain $\mathcal{D}$. The eigenfunctions of this operator, $\{\psi_k(t)\}$, denoted the generalized Morse wavelets are orthogonal functions and well concentrated with respect to the operator. We order them according to their concentration, that can be found as the eigenvalues of $\mathcal{P}_{\mathcal{D}}$ (see Olhede and Walden (2002)).
The wavelet transform is used to find the local signal energy content at any time and scale through the scalogram and we may find estimates with greatly reduced variability, as in for example Bayram and Baraniuk (1996)) by averaging over several orthogonal wavelets.
The generalized Morse wavelets are analytic complex functions, and so their Fourier transform is wholly supported on the positive frequencies. This has important consequences, namely that polarization of components with transient phase-shifts (Olhede and Walden (2003a, 2003b) may be detected, and the phase shift estimated. Polarization corresponds to a special relationship between the components, and can for exam-

ple be found in seismic data when the signals are dominated by surface waves – i.e. the Rayleigh and Love waves. Rayleigh waves exhibit elliptical polarization and retrograde particle motion in the vertical plane containing the direction of propagation: the radial/vertical plane. Love waves are characterised by horizontal motion perpendicular to the direction of propagation with no vertical motion: i.e. particle motion is all transverse.

Also the multiple estimates allow us to estimate the noise variance and in turn this gives us a method to estimate the uncertainty in the phase-estimate. We have applied the methodology to 3-component seismological data. The estimates are surprisingly accurate even in substantial noise.

References

Bayram, M and Baraniuk, R. (1996) Multiple window time-frequency analysis, in *Proc IEEE Int. Symp. Time-Freq. Time-Scale Analysis* Paris, France, 1996, 511–4.

Daubechies, I and Paul, T. (1988) Time-frequency Localization Operators - A Geometric Phase Space Approach: 2. The Use of Dilations. *Inverse Problems*, 4, 661–80,

Olhede, S.C. and Walden, A.T. (2002) Generalized Morse wavelets. *IEEE Trans. Signal Processing*, 50, 2661–2670.

Olhede, S.C. and Walden, A.T. (2003a) Polarization phase relationships via multiple Morse wavelets — Part I: Fundamentals. Proc. R. Soc. Lond. A., 459, 413-44.

Olhede, S.C. and Walden, A.T. (2003b) Polarization phase relationships via multiple Morse wavelets — Part II: Data analysis. Proc. R. Soc. Lond. A., 459, 641-57.