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Magnitude, phase, neighbour coefficients and thresholding

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The estimation of a deterministic signal in noise is a problem of importance, as often we wish to recognize coherent systematic behaviour characterised by the ordering of the data, when such behaviour is immersed in noise. Time-frequency/scale analyses have been used to construct procedures for this estimation, either based on decompositions in terms of a Gabor basis or a wavelet basis (see Donoho and Johnstone (1994)), where decomposition coefficients are thresholded, or set to zero, if they do not exceed a specified threshold. Wavelets are localised in time and frequency, and so the decomposition coefficients from a wavelet decomposition can represent signals that are mainly smooth but with some rapid variation in terms of a few coefficients. The signal is then estimated from the thresholded decomposition coefficients. For signals that can be represented in terms of a few decomposition coefficients, so called sparse signals, this procedure is found to give very good results.
The standard method proposed by Donoho and Johnstone (1994) has been modified and improved in many ways, and a simple but still very efficient modification of averaging over all possible time shifts corresponds to thresholding the undecimated wavelet transform coefficients (see Coifman and Donoho (1995)). Since the undecimated wavelet transform corresponds to a form of band-pass filtering, natural oscillatory components will manifest themselves in appropriate levels (frequency bands) and time intervals of the transform coefficients. We would prefer small values of the signal to be set to zero only where the amplitude is small and not due to the local phase of the signal, as this will introduce artifacts in the estimated phase of the signal. To alleviate these artifacts Olhede and Walden (2004) introduced analytic thresholding where a complex valued vector is constructed from the observed real vector using the discrete Hilbert transform. The real wavelet coefficients derived from the real vector are then thresholded according to the value of the amplitude found from the real and imaginary

wavelet coefficients constructed from decomposing the complex vector. This operation can also be considered as using information from neighbouring wavelet coefficients as the imaginary component is constructed from a weighted decaying average of the other vector components. For many signals significant improvements can be found from this procedure.

References

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