



Applications of the Renormalization Group Method in Upscaling Hydrological Parameters

D. T. Hristopulos

Department of Mineral Resources Engineering, Technical University of Crete, Chania, GR 73 100 (dionisi@mred.tuc.gr)

Hydrological parameters are characterized by significant spatial variability. The problem of upscaling refers to the estimation of coarse-grained parameters that incorporate fine-scale fluctuations at larger scales. In subsurface hydrology and petroleum engineering in particular, it is desired to determine coarse-grained parameters (such as hydraulic conductivity) that represent large-scale blocks, in a systematic way that incorporates the fine-scale heterogeneity and also respects the physical constraints imposed by the relevant transport equations. The resulting coarse-grained values can be used as input for numerically tractable simulations of groundwater flow and contaminant transport. Various upscaling approaches have been formulated that employ different measures of coarse-graining. This paper focuses primarily on stochastic upscaling methods and in particular the renormalization group (RG) approach. Certain aspects of spatial averaging methods are also discussed for reasons of comparison. The RG provides a powerful theoretical framework, more suitable for upscaling strong heterogeneity, than standard low-order perturbation expansions. Applications of RG methods in subsurface hydrology include the calculation of macroscopic transport parameters (e.g., effective and equivalent hydraulic conductivity, dispersion coefficients), and the estimation of anomalous exponents characterizing the dispersion of contaminants due to long-range conductivity correlations or broad (heavy-tailed) distributions of the groundwater velocity. The main ideas and assumptions underlying RG methods are reviewed and hydrological applications of the RG method from the literature are discussed. As a specific example of frequency-space RG application, it is shown that the exponential ansatz for the saturated hydraulic conductivity of porous media with isotropic, lognormal heterogeneity is obtained using a combination of the RG and the replica averaging method. The impact of finite domain sizes is also discussed, and the

concept of a fractional effective dimension is defined. Finally, a critical evaluation of the RG strengths and shortcomings for hydrological upscaling is presented.