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Universal scaling laws for distributions of energy, temporal and spatial characteristics in seismology

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To analyse seismicity (sequence of seismic events) it is necessary to determine an energy-spatial-temporal interval (E, ΔE; X, ΔX, Y, ΔY, Z, ΔZ; T, ΔT) – ESTI. But characteristics of seismicity are significantly inhomogeneous from one ESTI to another. In the same time it is possible to find relations between distributions of energy, temporal and spatial characteristics for different ESTI. This confirms the idea about existence of a common physical mechanism in earthquake generation process.

The universal scaling theory for energy, temporal and spatial characteristics is developed. It covers different approaches. Its application for v which is inter-event times Δt or a new space parameter Δd_{min} (the minimal distance from current seismic event to the nearest one in ESTI) is considered. The distribution of Δd_{min} determines the distance of the new seismic event from old ones. Thus this parameter is very important for seismic hazard estimation.

The main statement of the universal scaling theory is: a distribution F of a characteristic v is: $F(v|\text{ESTI}) = F_0(v/< v >')$ or for density of distribution $f(v|\text{ESTI}) = f_0(v/< v >')/< v >'$ (German, 2002, for inter-event times), where F_0 and f_0 are constant function and $< v >'$ is a scaling parameter. The calculation of the mean of v in ESTI $< v >$ with application of this statement shows that it is proportional to scaling parameter $< v >'$. Thus it is possible to use $< v >' = < v >$.

The analysis of the ratios $\Delta t/< \Delta t >$ and $\Delta d_{min}/< \Delta d_{min} >$ shows that they are well approximated by the Weibull distribution. In case that v is Δt or Δd_{min} it is possible to find the relation between $< v >$ and a number of events N_{ESTI} in ESTI: $< v > = 1/(WN^k_{ESTI})$, W = *const*, thus $< \Delta t > = \Delta T/N_{ESTI}$ and a new relation $< \Delta d_{min} > = C(\Delta L^2/N_{ESTI})^{d1}$ was determined, where C is constant for a region considered and $d_1 \approx 2.5$.

Such relations are very useful because N_{ESTI} is related with parameters of ESTI by means of the Gutenberg-Richter law, which in generalized form (with $\Delta E)$ is:

$$N_{ESTI} = A(X, Y, Z, T)\ E^{1-\gamma}\ (1 - [(E + \Delta E)/E]^{1-\gamma})\ \Delta L^d\ \Delta T\ =$$

$$= A(X, Y, Z, T)\ 10^{-bM}(1 - 10^{-b\Delta M})\ \Delta L^d\ \Delta T,$$

where $\Delta L = \Delta X = \Delta Y$ and M is magnitude, in most cases $\Delta E = const \cdot E$. Thus it is possible to write the scaling parameter through parameters of ESTI. The Gutenberg-Richter law determines the distribution of E (or $M)$ and the scaling parameter for it (application of the gamma distribution for E also shows a possibility for scaling).

Another approach for scaling (e.g. Bak et al., 2002) is: covering the spatial area of ESTI with a grid with cells of size ΔL_c x ΔL_c (new variants of temporal grid with cells size ΔT_c or for energy grid with ΔE_c are also considered). Let's assume a total number of cells is n and each i-th cell has N_i events and a characteristic v_i with a density of distribution $f_i(v)$, $< v_i > = 1/\ (WN_i^k)$, and let's assume $\varphi = 1/(W < N >^k)$ $(< N > = N_{ESTI}\ /n)$. In this case $R_i = N_i\ /< N >$ has also a stable distribution for each kind of grid with density function $f_r(r)$ (for spatial grid see also Corral (2003)). Therefore a mixture of density of distributions $f_i(v)$ in cells is $f_\Sigma(v)$ and

$$f_\Sigma(v) = \Sigma\{N_i\ f_i(v)\}/\ \Sigma N_i\ = \Sigma\{(N_i/< N_i >)f_i(v)\}/n =$$

$$= \Sigma\{(N_i/< N >)\ (WN_i^k)\ f_0(v\ WN_i^k)\}/n =$$

$$= W < N >^k\ \ \Sigma\{(N_i/< N >)^{k+1} f_0((N_i/< N >)^k\ W < N >^k\ v)\}\ /n =$$

$$= \varphi^{-1}\quad \Sigma\{R_i^{k+1}\ f_0(R_i^k v/\varphi)\}\ /n = \{\text{the last sum is just an average value or mean}\} =$$

$$= \varphi^{-1} \int\{r^{k+1}\ f_0(r^k\ v/\varphi)\ f_r(r)\}\ \mathrm{d}r = \varphi^{-1}\ f_{0C}(v/\varphi).$$

The last result means that it is possible to scale a mixture of distributions and in this case the scaling parameter is $\varphi = 1/(W < N >^k)$.

For $\Delta t \quad \varphi = \Delta T\ /< N >$ and for $\Delta d_{min}\ \varphi = C(\Delta L^2/< N >)^{d1}$. Again scaling parameter can be written thought parameters of ESTI and n (or size of grid) because $< N > = N_{ESTI}\ /\ n$.

The relation between Bak's et al. (2002) scaling $f(v|\text{ESTI}) = g_0(v/v_s)/v$, v_s is the scaling parameter, and the universal scaling theory is also determined:
$f_\Sigma(v) = \varphi^{-1}\ f_{0C}(v/\varphi) = v^{-1}\ (v/\varphi)\ f_{0C}(v/\varphi) = v^{-1}\ f_{0B}(v/\varphi)$,
thus $f_{0B}(v/\varphi) = v \quad f_{0C}(v/\varphi)$. Now it is clear, that to obtain Δt^{-1} for v is Δt it is not necessary to use Omori law.

The statistical analysis of scaled variables allows to obtain the Cox's accelerated life model. According to it a standard deviation of v for any $< v >$ is a constant and $<\log v> = \log(< v >) + const$.

This relation is useful for checking of possibility of scaling and for determination of the scaling parameter properties by means of least square method application.

All relations are demonstrated for Δt and Δd_{min} for some seismic events catalogues (earthquakes and rockbursts).