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Determination of the parameters of the abridging Molodensky formulae providing the best horizontal fit

G. Molnár and G. Timár

Space Research Group, Dept. of Geophysics, Eötvös University of Budapest, Hungary

(spacerg@sas.elte.hu / Fax: +36 1 3722927 / Phone: +36 1 2090555 6651)

Introduction

In the geographic information systems (GIS), one of the most important tasks is to fit data and maps, given in different projection systems, into a common base. The transformation of the coordinates can be done by several methods. One of them is to convert the grid coordinates of the source data to geodetic ones, then convert them to the geodetic coordinates of the target system (datum transformation) and calculate the grid coordinates in the target system. The mentioned datum transformation can be done by the Molodensky formulae (Molodensky *et al.*, 1960; Badekas, 1969) or by using the Burša-Wolf transformation (Burša, 1962; Wolf, 1963).

The abridging Molodensky formulae use the relative location of the geometric centres of the source and target ellipsoids (datums). The parameters are the three components of the vector, connecting the ellipsoid centres. The aim of the present work is to estimate the optimal parameter set, providing the best horizontal fit using a set of basepoints with given coordinates in both systems.

Method and results

The horizontal components of the abridging Molodensky formulae are (DMA, 1990):

$$\Delta\Phi" = \frac{-dX\sin\Phi\cos\Lambda - dY\sin\Phi\sin\Lambda + dZ\cos\Phi + (a\cdot df + f\cdot da)\sin 2\Phi}{M\sin 1"}$$

$$\Delta\Lambda" = \frac{-dX\sin\Lambda + dY\cos\Lambda}{N\cos\Phi\sin 1"}$$

$\Delta\Phi"$ and $\Delta\Lambda"$ are the differences between the coordinates of a basepoint in the two systems in arc second units, a and f are the semimajor axis and the flattening of the source ellipsoid, respectively, and *da* and *df* are the semimajor axis and flattening of the source and target ellipsoids. M and N are the curvatures in the prime meridian and in the prime vertical on the source ellipsoid. Using the basepoint set with the coordinates both in the source and the target system, the differences between the observed and the calculated coordinates should be minimized, as follows:

$$\sum_{i=1}^{N}\left(\Phi_T^{(i)}-\Phi_S^{(i)}+\Delta\Phi''^{(i)}\left(\Phi_S,\Lambda_S\right)\right)^2+\sum_{i=1}^{N}\left(\cos\Phi_S^{(i)}\cdot\left(\Lambda_T^{(i)}-\Lambda_S^{(i)}+\Delta\Lambda''^{(i)}\left(\Phi_S,\Lambda_S\right)\right)\right)^2=\min$$

where the 'S' lower index indicates the source coordinates and the 'T' indicates the target ones. These values can be calculated using the Molodensky formulae as a function of the geodetic coordinates. To get the optimum in a planar system instead of the geodetic system, the longitude difference is scaled by $\cos(\Phi)$. The condition of the minimum is that the partial derivative of the square sums of the differences in the first two equations, by the parameters, should be all zeroes.

Doing the partial derivations and using the value C=*a*·*df*+*f*·*da*, the above equation can be expressed in the following matrix form:

$$Ax=b$$

where the elements of the (symmetric) matrix **A** and the vector **b** are:

$$A_{11}=\sum_{i=1}^{N}\left[\left(\frac{\sin\Phi^{(i)}\cos\Lambda^{(i)}}{M^{(i)}\sin 1''}\right)^2+\left(\frac{\sin\Lambda^{(i)}}{N^{(i)}\cos\Phi^{(i)}\sin 1''}\right)^2\right]$$
$$A_{12}=\sum_{i=1}^{N}\left[\frac{\sin^2\Phi^{(i)}\sin\Lambda^{(i)}\cos\Lambda^{(i)}}{\left(M^{(i)}\sin 1''\right)^2}-\frac{\sin\Lambda^{(i)}\cos\Lambda^{(i)}}{\left(N^{(i)}\cos\Phi^{(i)}\sin 1''\right)^2}\right]$$
$$A_{13}=\sum_{i=1}^{N}\left[\frac{-\sin\Phi^{(i)}\cos\Phi^{(i)}\cos\Lambda^{(i)}}{\left(M^{(i)}\sin 1''\right)^2}\right]$$
$$A_{22}=\sum_{i=1}^{N}\left[\left(\frac{\sin\Phi^{(i)}\sin\Lambda^{(i)}}{M^{(i)}\sin 1''}\right)^2+\left(\frac{\cos\Lambda^{(i)}}{N^{(i)}\cos\Phi^{(i)}\sin 1''}\right)^2\right]$$
$$A_{23}=\sum_{i=1}^{N}\left[\frac{-\sin\Phi^{(i)}\cos\Phi^{(i)}\sin\Lambda^{(i)}}{\left(M^{(i)}\sin 1''\right)^2}\right]$$
$$A_{33}=\sum_{i=1}^{N}\left[\left(\frac{\cos\Phi^{(i)}}{M^{(i)}\sin 1''}\right)^2\right]$$

$$b_1 = \sum_{i=1}^{N} \left[\frac{-\Delta\Phi^{(i)} \sin\Phi^{(i)} \cos\Lambda^{(i)}}{M^{(i)} \sin 1''} + \frac{C \sin 2\Phi^{(i)} \sin\Phi^{(i)} \cos\Lambda^{(i)}}{\left(M^{(i)} \sin 1''\right)^2} - \frac{\Delta\Lambda^{(i)} \sin\Lambda^{(i)}}{N^{(i)} \cos\Phi^{(i)} \sin 1''} \right]$$
$$b_2 = \sum_{i=1}^{N} \left[\frac{-\Delta\Phi^{(i)} \sin\Phi^{(i)} \sin\Lambda^{(i)}}{M^{(i)} \sin 1''} + \frac{C \sin 2\Phi^{(i)} \sin\Phi^{(i)} \sin\Lambda^{(i)}}{\left(M^{(i)} \sin 1''\right)^2} + \frac{\Delta\Lambda^{(i)} \cos\Lambda^{(i)}}{N^{(i)} \cos\Phi^{(i)} \sin 1''} \right]$$
$$b_3 = \sum_{i=1}^{N} \left[\frac{\Delta\Phi^{(i)} \cos\Phi^{(i)}}{M^{(i)} \sin 1''} - \frac{C \sin 2\Phi^{(i)} \cos\Phi^{(i)}}{\left(M^{(i)} \sin 1''\right)^2} \right]$$

In these matrices and vectors, all the coordinates, the M and N values are interpreted in the source system. This is an inhomogeneous linear equation system, whose solution is

$$x = A^{-1}b$$

where $\mathbf{A}^{-1}$ is the inverse of the matrix **A.** The solution vector **x** contains the *dX*, *dY* and *dZ* parameters. In the practice, the parameters can be easily determined by the Cramer rule.

Conclusions

If a basepoint system is given with their coordinates in two independent geodetic system, the above given equations are capable to estimate the parameters of the Molodensky formulae. In the GIS practice, it enables to define the datum transformation parameters properly between the two used (the source and the target) datums. This solution is not necessarily precise in three dimensions but the estimated parameters provide better horizontal fit than the ones, which were derived from the three-dimensional locations. These parameters are usually not precise enough for geodetic purposes but useful in the GIS applications.

References

Badekas, J. (1969): Investigations related to the establishment of a world geodetic system. *Report 124*, Department of Geodetic Science, Ohio State University, Columbus.

Burša, M. (1962): The theory for the determination of the non-parallelism of the minor axis of the reference ellipsoid and the inertial polar axis of the Earth, and the planes of the initial astronomic and geodetic meridians from the observation of artificial Earth satellites. *Studia Geophysica et Geodetica* **6**: 209-214.

DMA, Defense Mapping Agency (1990): Datums, Ellipsoids, Grids and Grid Reference Systems. DMA Technical Manual 8358.1. Fairfax, Virginia, USA.

Molodensky M. S.–Eremeev, V. F.–Yurkina, M. I. (1960): Metody izucheniya vnesnego gravitatsionnogo polya i figuri Zemli. *Tr. CNIIGAiK [Moscow]*, **131**.

Wolf, H. (1963): Geometric connection and re-orientation of three-dimensional triangulation nets. *Bulletin Géodésique* nr. **68**:165-169.