



## **Level sets, implicit function theorem, and their application in gravity field modelling (spherical and ellipsoidal examples)**

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There are many problems in geodesy, and especially in gravity field modelling, which can be formulated as the problem of finding level sets of vector spaces. In all those problems implicit function theorem is an efficient tool for finding singularity points or critical points of the vector space where the level sets doesn't exist. In this paper we have constructed variety of problems including following problems in terms of the problem of finding the level sets:

1. Computation of radius of Bjerhammar sphere.
2. Computation of semi-major and semi-minor axis of Somigliana-Pizzetti level ellipsoid, and
3. Computations of spherical and ellipsoidal Bruns formulas.

In all the aforementioned applications, and others which are included in the paper, implicit functions theorem has been used for finding the singularity points.

The application of level sets in geodesy backs to the historical mappings of the earth's topography in terms of contour lines. Given the topography of the earth in terms of the functional  $z = z(x, y)$  its graph is  $f(x, y, z) = 0$ , and its contour line  $x = x(c, y)$  or  $y = y(x, c)$ . The problem of finding the contour line  $x = x(c, y)$  or  $y = y(x, c)$ , can be stated as follows:

“Given” the functional  $z = z(x, y) = c$ ;

“Find” the functional  $x = x(c, y)$  or  $y = y(x, c)$ , where  $x = x(c, y)$  or  $y = y(x, c)$  are called the shape function of the level sets.

The problem of geoid computation can also be viewed as the problem of finding level sets. If the actual potential field of the earth be given by the functional  $w = w(\lambda, \phi, h)$  and the geoid's potential value by  $w_0$ , then the problem of geoid computations can be presented as:

“Given” the function  $w = w(\lambda, \phi, h) = w_0$ ;

“Find” the function  $h = h(\lambda, \phi, w_0)$ , where  $h = h(\lambda, \phi, w_0)$  is called the shape function of the geoid.