



Analysis of different numerical methods of solving 2D advection-diffusion transport equation

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The problem under consideration is the mixing of passive, dissolved and conservative substance released into the river. Most natural rivers are shallow compared with their width and length so the process (except the initial distance) can be represented by depth-averaged, two dimensional advection-diffusion, partial differential equation:

$$\frac{\partial c(\mathbf{x}, t)}{\partial t} = \nabla (\mathbf{D}(\mathbf{x}, t) \cdot \nabla c(\mathbf{x}, t)) - \nabla (\mathbf{v}(\mathbf{x}, t) \cdot \nabla c(\mathbf{x}, t))$$

where:

- t – time,
- $\mathbf{x} = (x, y)$ – longitudinal and transverse coordinates,
- $c(\mathbf{x}, t)$ – solute concentration,
- $\mathbf{v}(\mathbf{x}, t)$ – velocity field,
- $\mathbf{D}(\mathbf{x}, t)$ – dispersion tensor.

To solve the equation one needs to know the initial concentration field and the boundary conditions, the velocity field and the dispersion coefficients. With general boundary conditions the equation does not have an exact analytical solution and therefore problem must be solved numerically. Solving 2D advection-diffusion equation numerically we can encounter four major problems: numerical diffusion, numerical oscillation, limitation of CPU time and disk space and way of proper setting the boundary conditions. Various computational algorithms for the simulation of the transport equations in the depth-averaged form have been widely used for many years now. Here we consider finite difference methods. It is important to choose the

best scheme depending on the problem to be solved and the necessary accuracy and anticipated simulation time should be taken into consideration.

A brief review of the performance of several numerical methods (like Upwind, Crank-Nicolson, Alternating Direction Implicit methods) has been provided herein. Numerical methods are tested for a simple case and stability, accuracy and efficiency of algorithms have been studied. Tested schemes will be used to create the River Mixing Model to simulate 2D river mixing process.