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Spatiospectral concentration and analysis on the sphere

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It is often advantageous to investigate the relationship between two geophysical datasets in the spectral domain by calculating their admittance and coherence spectra, for instance when the coherence between gravity anomalies and topography is inverted for elastic thickness. While powerful Cartesian techniques exist for localized spectral and cross-spectral estimation, the inherent sphericity of planetary bodies sometimes necessitates an approach based in spherical coordinates. Direct localized spectral estimates on the sphere can be obtained by multiplying a dataset by a suitable windowing function (or data taper) and expanding the resultant field in spherical harmonics. Bandlimited functions that are optimally concentrated in space, or, alternatively, spacelimited functions that are optimally concentrated in the spectral domain are required for the spatiospectral localization of geophysical processes. We have solved Slepian's concentration problem on the sphere to find an orthogonal basis of spectrally concentrated spherical harmonic expansions that are also optimally concentrated inside an arbitrary region on the unit sphere. We show how such functions can be found either in the Fourier domain by an eigenvalue analysis of a kernel reminiscent of a spatially truncated Kronecker delta function, or in space by diagonalizing a Fredholm integral equation kernel consisting of a bandlimited Dirac delta function. Our functions are asymptotically self-similar when a Shannon number, or area-bandwidth product, is kept invariant. This is in complete analogy with Slepian's solutions on the real line or their extensions to the time-frequency or space-frequency planes. The new class of functions is both easily calculated and lends itself to practical problems of spherical spectral analysis. When the spectral coefficients of an input data field are governed by a white stochastic process, the expectation of the localized power spectrum using our data tapers (averaged over all possible realizations of the random variables) is shown to be nearly unbiased. For the typical geophysical situation in which only one realization of such a process is available, the weighted average of the spectra obtained from

multiple data tapers approaches the expected spectrum with increasing Shannon number. When analyzing the relationship between two fields on the sphere, our method allows for the robust localized estimation of the spectral admittance and coherence functions in a manner as optimal as it is computationally efficient, as we show by example.