



Pre-bifurcational noise-dependent phenomena as diagnostic instrument for revealing bifurcations in geophysical systems

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1. Introduction

This paper studies three fundamental phenomena, which could be observed in geophysical systems near the threshold of a bifurcation: the pre-bifurcation noise amplification, the pre-bifurcation rise and saturation the fluctuations' correlation time and the noise-dependent hysteresis.

In the case of a slowly (quasi-stationary) changing control parameter, the variance of the forced fluctuations increases when approaching the threshold of bifurcation. The analysis of the pre-bifurcation noise amplification, performed in paper [1], is based on linear theory, which demonstrates an unlimited growth of fluctuations in the immediate vicinity of the bifurcation point. According to [1], pre-bifurcation noise amplification might be an effective diagnostic instrument for nonlinear systems (called "noisy precursor" of bifurcation). A nonlinear saturation of fluctuations in the vicinity of bifurcation point was studied in the papers [2]. A new non-linear fluctuation phenomenon of the pre-bifurcation rise of the fluctuations' correlation time followed with saturation in the immediate vicinity of the bifurcation threshold has been recently performed in [3].

At fast bifurcation transitions ("dynamical bifurcations") another phenomenon occurs – the noise-dependent hysteresis. This phenomenon was first mentioned in [4] where it was shown that under forward bifurcation transitions, the time-delay between the break-away point and the critical bifurcation value is proportional to the square root of the rate of the control parameter change. The phenomenon of hysteresis was observed

experimentally for a transition from laminar to vortex shedding flow in soap films [5].

The main goal of this contribution is to attract the attention of geophysicists to these phenomena and thereby to stimulate searches for specific mechanisms that might be responsible for the pre-bifurcation noise amplification and the noise-dependent hysteresis in Geophysics.

2. Core

We study pre-bifurcation noise amplification in nonlinear systems subject to pitchfork bifurcations and period doubling bifurcations which are manifested in the doubling of stable equilibrium states. These bifurcations could be observed in geophysical systems [6, 7]. Our theoretical estimates of both the linear growth and the nonlinear saturation of the bifurcations are in good agreement with our results from numerical simulations. We show that in the saturation mode, the fluctuation variance σ_x^2 is proportional to the standard deviation of the external noise σ_η , whereas in the linear mode, σ_x^2 is proportional to the noise variance σ_η^2 .

The phenomenon of pre-bifurcation correlation time rise accompanies the phenomenon of pre-bifurcation amplification of fluctuation intensity, and, similarly to the latter, experiences saturation in the vicinity of the bifurcation point. Away from the bifurcation point the correlation time k_c grows inverse proportional to the parameter α , which characterizes the closeness to the bifurcation threshold, whereas immediately at the threshold, the correlation time reaches a saturation at a level which is inversely proportional to the noise standard deviation σ_η .

It is demonstrated that the phenomena of the pre-bifurcation noise amplification and the pre-bifurcation correlation time rise are more pronounced in the case of a slow transition through the bifurcation point. The amplification of fluctuations in this case makes it easier to form a symmetric probability of the final equilibrium states.

In contrast, for a fast transition through the bifurcation point, the effect of amplification is much less pronounced. Under backward and forward passages through the bifurcation point, a loop of noise-dependent hysteresis emerges here. We find that for a fast transition through the bifurcation point, the probability symmetry of the final equilibrium states is destroyed.

These phenomena are studied as prospective indicators of bifurcations ("noisy precursor") in nonlinear geophysical systems [5-7].

3. Discussion

i) The analysis is performed for a specific case of the period-doubling bifurcation and the pitchfork bifurcation, however, similar regularities are expected to be valid

also for other kinds of bifurcations such as pre-generation noise amplification near the Landau-Hopf bifurcation.

ii) As was pointed out by Wiesenfeld [1], the phenomenon of pre-bifurcation noise amplification as well as weak signal amplification at the bifurcation threshold might be an effective tool for revealing bifurcations in nonlinear systems. Experimental manifestation of pre-bifurcation noise enhancement is described in [8] dealing with a period doubling bifurcation in a semiconductor laser. Another example of pre-bifurcation noise amplification is given in [9] for fluctuations of light in a ring cavity with a nonlinear absorbing medium.

iii) A significant growth of pre-bifurcation fluctuations seems to be the main reason for the bad predictability for a nonlinear system transiting through the bifurcation point. That is why fast transitions through the bifurcation point are more predictable than slow ones. This approach looks to be prospective for the analysis of different nonlinear geophysical systems in which period-doubling bifurcations and pitchfork bifurcations take place, e.g. the variation of the North Equatorial Current bifurcation [6] or the Kuroshio bifurcation [7] as well as in many other nonlinear geophysical systems.

References

1. Wiesenfeld K, Phys. Rev.A 32, 1744 (1985).
2. Kravtsov Yu. A, Surovyatkina E. D., Physics Letters A 319, 348 (2003).
3. Surovyatkina E. D., Physics Letters A 329, 169 (2004).
4. Kapral, R. and Mandel, P., Phys. Rev. A 32, 1076 (1985).
5. Horvath, V., Cressman., J, Goldburg, W., Xiao-Lun Wu: Hysteretic transition from laminar to vortex shedding flow in soap films. Advances in Turbulence VIII. Proc. 8th European Turbulence Conference, Barcelona, Spain, edited by C. Dopazo, 23 (2000).
6. Kim, Y. Y., T. Qu, T. Jensen, T. Miyama, H. Mitsudera, H.-W. Kang, and A. Ishida, J. Geophys. Res., 109, C03040, doi:10.1029/2003JC002013 (2004).
7. Joyce T. M., Dunworth-Baker J., J. Geophys. Lett. 30, 2, 1043, doi:10.1029/2002GL015225 (2003).
8. Lamela H., Perez S., Carpintero G., Optics Letters 26, 69 (2001).
9. Garcia-Ojalvo J., Roy R., Phys. Lett. A 224, 51(1996).