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# The ‘analytic’ vector in stochastic time-frequency analysis

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The analytic signal was introduced by Gabor in 1946 and is used unambiguously to define the amplitude (magnitude) and phase of a real signal in continuous time. Its finite-length discrete version is the ‘analytic’ vector, the complex-valued vector with real part equal to a specified real-valued vector, and imaginary part equal to the result of applying the discrete Hilbert transform matrix to the specified vector. We discuss a successful use of the analytic vector. It is assumed observed signals are potentially noisy, and thus the analytic extensions of both the deterministic and stochastic components need to be interpretable and tractable. Our application is to time-frequency analysis of multicomponent signals. We seek to determine the energy of each component by frequency at each time. Multicomponent signals are notoriously hard to deal with in time-frequency analysis. Our approach involves analytic vectors in tandem with the maximal overlap discrete wavelet packet transform (MODWPT).

We discuss firstly the MODWPT, which is simply the undecimated version of the standard discrete wavelet packet transform (DWPT). The DWPT is simply a generalization of the discrete wavelet transform which at level $j$ of the transform partitions the frequency axis into $2^j$ equal width frequency bands.

For the time-frequency analysis of multicomponent signals (Olhede & Walden, 2005) analytic vectors are created before and after a demodulation step, followed by a projection onto the time-frequency plane so that, as closely as possible, each component of the signal contributes exclusively to a different ‘tile’ in the MODWPT tiling of the time-frequency plane, and at each time instant the contribution to each tile definitely comes from no more than one component. A single reverse demodulation is then applied to all projected detail components. The resulting instantaneous frequency of each

component in each tile is not constrained to a set polynomial form in time, and is readily calculated, and when weighted by the local energy yields the corresponding Hilbert energy spectrum which tells us about the energy of each component by frequency at each time.

The application depends on the deterministic and stochastic properties of the analytic vector, and it is the joint behaviour of the two, at any particular time, that ensures the utility of the methodology.

Reference


1. OLHEDE, S. C. & WALDEN, A. T. (2005) A generalized demodulation approach to time-frequency projections for multicomponent signals. *Proc. R. Soc. Lond. A*, under revision.