



## **Geodynamics of water flows in the Dead Sea basin: A proposed model for analysis of stratified, rotating and compressible liquid**

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The Dead Sea with a total area of 1,049 km<sup>2</sup> is located at the boundary between Israel and Jordan. It is a lowest place at the Earth (level of 2004 is about 418 m below the Mediterranean Sea level) with a unique physical-chemical water composition. Thermal and pressure characteristics in this area are characterized by strong inhomogeneity [1]. A density of water in vertical direction changes from 1150 kg/m<sup>3</sup> at the Dead Sea surface up to 1450 kg/m<sup>3</sup> at the depth of 310-330 m. Such a density gradient in a few hundreds times exceeds the normal water density gradient in the Earth's oceans and seas. The most significant source of this density gradient is a constant inflow of mineral substance from the great depth (apparently, from upper mantle). Geodynamically, density stratification is the most significant factor comparing with other types of stratifications. Thus, the Dead Sea basin maybe considered as a distinctive native polygon of vertically stratified liquid. For analysis of water flows geodynamics in the Dead Sea basin maybe used solutions obtained for motion of stratified liquid for different initial boundary values. Taking into account a very high modern rate of changing physical, chemical and geological characteristics of this unique basin, the development of geodynamical predictable model of the area could have a supreme importance.

Let's consider a motion of stratified, rotating and compressible liquid (*SRCL*) in the Cartesian coordinates  $(x_1, x_2, x_3)$  which is rotating together with the *SRCL*. Let's assume that *SRCL* rotating is being around axis  $Ox_3$  and Coriolis vector  $\mathbf{f} = \{0, 0, f\}$ , where  $f$  is the double angle velocity rotating. *SRCL* is stratified along the axis  $Ox_3$ ,

i.e. its density in undisturbed condition depends on  $x_3$ ,  $\rho_0 = \rho_0(x_3)$ . We assume below that  $\rho_0(x_3) = Ae^{-2\beta x}$ ,  $A > 0$ ,  $\beta > 0$ . Small *SRCL* motions under influence of gravity acceleration without any external forces maybe described as [2]:

$$\left. \begin{aligned} \rho_0(x_3) \frac{\partial \mathbf{v}}{\partial t} + \rho_0(x_3) [\mathbf{f}, \mathbf{v}] + \nabla p + \mathbf{e}_3 \rho_1 g &= 0 \\ \frac{\partial \rho_1}{\partial t} + (\mathbf{e}_3 \cdot \mathbf{v}) \rho'_0(x_3) \operatorname{div} \mathbf{v} &= 0 \\ \frac{\partial \rho_1}{\partial t} = \frac{1}{c^2} \frac{\partial p}{\partial t} + \rho_0(x_3) \omega_0^2(x_3) \frac{(\mathbf{e}_3 \cdot \mathbf{v})}{g} \end{aligned} \right\}, \quad (1)$$

where  $g$  is the gravity acceleration,  $\mathbf{v} = \{v_1, v_2, v_3\}$  is the vector of *SRCL* particles motion,  $\rho_1$  is the changing of *SRCL* density caused by its motion,  $c$  is the acoustic velocity,  $p$  is the dynamic pressure and  $\mathbf{e}_3$  is the ort of axis  $Ox_3$  and  $[\mathbf{f}, \mathbf{v}]$  denotes the vector product between two vectors. Value  $\omega_0^2 \geq 0$  is the quadrate of Vyasyalay-Brent frequency. A condition  $\omega_0^2 \geq 0$  means absence of convective motions and stability of the considering density distribution  $\rho_0(x_3)$  in the *SRCL*. After several bulky transformations Eq. (1) in [2] was reduced to the following type:

$$\frac{\partial^2}{\partial t^2} \left[ \frac{\partial^2 u}{\partial t^2} - c^2 \Delta_3 u + (f^2 + \beta^2 c^2) u \right] - \left[ \omega_0^2 c^2 \Delta_2 u + f^2 c^2 \frac{\partial^2 u}{\partial x_3^2} - \beta^2 c^2 f^2 u \right] = 0, \quad (2)$$

where  $u$  is a scalar function on which unknown functions  $\mathbf{v}$ ,  $\rho_1$  and  $p$  depend.

A generalization of Eq. 2 was investigated in [3]. Let's consider the following problem

$$\left. \begin{aligned} L(t, D_t, D_x) u(t, x) &= \frac{\partial}{\partial t^2} \left[ \frac{\partial^2 u(t, x)}{\partial t^2} - \sum_{i,j=1}^3 \frac{\partial}{\partial x_i} \left( A_{ij}(t, x) \frac{\partial u(t, x)}{\partial x_j} \right) \right] \\ + \sum_{i=1}^3 B_i(t, x) \frac{\partial u(t, x)}{\partial x_i} + C(t, x) u(t, x) &+ \sum_{i,j=1}^3 \frac{\partial}{\partial x_i} \left( a_{ij}(t, x) \frac{\partial u(t, x)}{\partial x_j} \right) \\ + \sum_{i=1}^3 b_i(t, x) \frac{\partial u(t, x)}{\partial x_i} + d(t, x) u(t, x) &= f(t, x), \quad (t, x) \in [0, T] \times \Omega \end{aligned} \right\}, \quad (3)$$

where  $A, B, C, a, b, d$  are given smooth enough functions (in application, see Eq. (2)), they depend on  $c, f, b$  and  $\omega_0$ ) and  $\Omega$  is a bounded set in  $R^3$ ,

$$u(t, x)|_{x \in \partial \Omega} = 0, \quad t \in [0, T], \quad (4)$$

$$\frac{\partial^k u(t, x)}{\partial t^k} \Big|_{t=0} = u_k(x), \quad k = 0, \dots, 3, \quad x \in \Omega. \quad (5)$$

Eqs. (4) and (5) present the Dirichlet boundary condition and the Cauchy initial conditions, respectively. Thus, Eqs. (3)-(5) maybe examined as a foundation for development of geodynamical model of the *SRCL* in the Dead Sea basin. An availability of such physical parameters as gravity acceleration, water density and pressure, temperature and acoustic velocity allow us to create a solvable system of equations and utilize it for prediction of changing geodynamical environments in the Dead Sea area.

### References

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