A comparison of the "patched-conics approach" and the restricted problem for swing-bys

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In the present paper we study maneuvers that uses the giant planets of the Solar System as the body for the close approach. The goal is to simulate a large variety of initial conditions for those orbits and classify them according to the effects caused by the close approach in the orbit of the spacecraft. The main goal is to perform a detailed comparison between the results obtained using the well-known planar restricted circular three-body problem and the "two-body problem" approximation known as "patched-conics." For the restricted problem model, the equations are regularized (using Lemaître's regularization), so it is possible to avoid the numerical problems that come from the close approach with Jupiter.

To perform this task, the following procedure is used:

i) Using the values of the energy $(E-)$ and angular momentum $(C-)$ before the close approach obtained by the numerical integration of the restricted problem, the semi-major axis (a) and the eccentricity (e) of the keplerian orbit before the passage is obtained. Those values are assumed to be the initial values for both maneuvers, the one using the "patched-conics" model and the one using the restricted model;

ii) Starting from this orbit, the variation in energy and angular momentum given by the "patched-conics" model is obtained;

iii) Then, the energy after the passage is obtained by $E_{+PC} = E- + \Delta E$, as well as the angular momentum $C_{+PC} = C- + \Delta C$;

iv) The semi-major axis and the eccentricity of the keplerian orbit that follows the passage using the "patched-conics" model is obtained, as well as the same quantities based in the restricted problem;

v) Finally the variations of all the variables involved are calculated.

The difference in the variation in energy, between the two models, are shown in several figures. The results showed that the differences between the two models:

- decrease in magnitude when the periapsis distance increase, what is expected since the general effects of the swing-by decrease with this variable;

- the most negative values for this variable are concentrated close to Ψ = 270° for the smallest values of the velocity of approach, so the “patched-conics” model underestimated the variation in energy close to the maximum effect of the Swing-By;

- the most positive values for this variable are concentrated in the interval 1.5 $<$ V_∞ $<$ 2.0 and 210° $< \Psi <$ 240°;

- the typical values for the energy variation have an order of magnitude 1.0, so the maximum differences between the two models (about 0.05) are in the order of 5 %. A detailed plot with the differences expressed in percentage is not shown, because values of the variation in energy close to zero generate values too large for the percentage error;

- the influence of those differences in the semi-major axis, eccentricity and angular momentum before and after the passage were also studied. The detailed results are not shown here due to the limitation of space, but they are of the same order of magnitude, except for situations where the energy is small. In those cases, small alterations of the variation in energy causes a large variation in the semi-major axis, eccentricity and angular momentum of the orbits involved and in the excess velocity and flight path angle in the crossing points with the Earth’s orbit. A detailed study for the case R_p = 1.1 R_j showed that trajectories with energy before or after the passage small enough to cause an error in the semi-major axis greater than 0.1 canonical units (10% of the Sun-Jupiter distance) occur in 5% of the trajectories calculated. There are also trajectories with errors in semi-major axis of several hundreds of canonical units. In those situations, the use of more complex models, like the one shown here is justified.