Gravitational capture by the major primary in the restricted three-body problem

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The ballistic gravitational capture is a characteristic of some dynamical systems in celestial mechanics, as in the restricted three-body problem that is considered in this paper. The basic idea is that a spacecraft (or any particle with negligible mass) can change from a hyperbolic orbit with a small positive energy around a celestial body into an elliptic orbit with a small negative energy without the use of any propulsive system. The force responsible for this modification in the orbit of the spacecraft is the gravitational force of another body involved in the dynamics. In this way, this force is used as a zero cost control, equivalent to a continuous thrust applied in the spacecraft. The present paper study in some detail the ballistic gravitational capture performed by the first primary in a three body system. Analytical equations for the forces involved in this maneuver are derived to estimate their magnitude and to show the best directions of approach for the maneuver.

The model used to study this problem is the planar restricted three-body problem. Several systems of primaries are considered for the simulations shown in this paper. The standard canonical system of units is used, in which the unit of distance is the distance between M_1 (the Earth) and M_2(Moon); the angular velocity of the motion of M_1 and M_2 is set to unity; the mass of the smaller primary (M_2) is given by $\mu \mathrm{yvo} \mu \mathrm{xr} \mu \mathrm{yp}$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_2 is (1-μ); the unit of time is defined such that the period of the motion of the two primaries is 2π and the gravitational constant is unity.

The paper also shows an explanation of the phenomenon based in the calculation of the forces involved in the dynamics as a function of time and in their integration with respect to time. Analytical equations are derived to study this problem under the assumption of radial motion, which leads to the derivation of an equation that estimates the reduction of C_3. Then, the forces acting on the ballistic gravitational capture problem are obtained in closed forms. There are two forces that act as disturbing forces in the direction of motion: the gravitational force due to the Moon and the centrifugal force. These forces can decelerate the spacecraft, working opposite to its motion. This is equivalent to applying a continuous propulsion force against the motion of the spacecraft. In the radial direction the gravitational force due to the Moon and the centrifugal force work in opposite directions, but the resultant force always works against the motion of the spacecraft.