



# Working Models for the Gravity Field of Phobos

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## Abstract

Working models for the gravity field of Phobos have been developed using the latest available Phobos shape model. Results from analytical and numerical approaches are presented. Spherical harmonic coefficients of the gravity field have been obtained up to degree and order 6. A higher-resolution gravity field model is obtained from numerical integrations. On the basis of the new models we propose to study the dynamical environment on the surface of Phobos, taking into account gravity, centrifuge, as well as tidal forces, which will be applied to studies of the landing site for the up coming ‘Phobos-Grunt’ mission. Our models will also be useful to study spacecraft motion near Phobos.

## 1. Introduction

Little is known about the origin and evolution of Phobos, the larger of the two Martian satellites. While detailed shape models of Phobos are available [4, 5], limited data exist for the satellite’s gravity field, with even  $GM$  still suffering from large uncertainties.

However, the knowledge of its gravity field is not only important for the study of the surface morphology and inner structure, but also crucial for navigation of spacecrafts approaching Phobos. Moreover, a landing mission like ‘Phobos-Grunt’ requires detailed local gravity and surface acceleration information for selection of a safe landing site.

Fortunately, useful working models for the gravity field may be derived from shape models. For a homogeneous body, the spherical harmonic coefficients of its gravity field can be derived analytically from its shape, as demonstrated by Balmino, 1994 [1]. The gravity field can also be

calculated numerically by approximating the body using an assembly of small volume elements.

A new shape model has recently been developed using mainly the data from SRC (Super Resolution Channel) of the HRSC (High Resolution Stereo Camera) on board ESA’s Mars Express mission [4]. The availability of the new shape model motivated us to undertake a new effort of updating previous gravity field models using both analytical and numerical approaches.

## 2. Method

The gravity field of a celestial body can be expressed using complex spherical harmonic coefficients as

$$V(r, \varphi, \lambda) = \frac{GM}{r} \sum_{l=0}^{\infty} \left(\frac{R}{r}\right)^l \sum_{m=-l}^l K_l^m Y_l^m(\varphi, \lambda) \quad (1)$$

In the case of Phobos, by assuming a homogeneous density, according to [1],  $K_l^m$  can be derived by integration over the shape.

$$K_l^m = \frac{4\pi\rho R_0^{l+3}}{(2l+1)(l+3)MR^l} \iint (1+S)^{l+3} Y_l^{m*} \sin\varphi' d\varphi' d\lambda' \quad (2)$$

Here  $S$  describes the deviation of the actual radius from a reference radius in the expression of spherical harmonic functions. Therefore, we can obtain the coefficients of the gravity field directly from the coefficients of the shape model.

Alternatively, according to Newton’s law of gravitation and the principle of superposition, the gravitational potential of a body with finite volume is

$$V = G \iiint \frac{\rho(x, y, z)}{r(x, y, z)} dx dy dz \quad (3)$$

By dividing the whole body of Phobos into small identical elements, the gravitational potential at  $\bar{R}$  can be approximately expressed as

$$V(\bar{R}) \approx \rho \Delta v \sum_i \frac{1}{|\bar{R} - \bar{r}_i|} \quad (4)$$

where  $\Delta v$  is the volume of one element.

To investigate the dynamical environment of Phobos, we adopted the definition of dynamic height given in [3] as

$$H_d = \frac{W_1 - W_0}{g_r} \quad (5)$$

$W_1$  and  $W_0$  are the measured and reference potential energies which consist gravitational potential, tidal potential and rotational potential.  $g_r$  is the reference gravity force.

### 3. Results

Spherical harmonic coefficients of the gravity field have been obtained up to degree and order 6 using the analytical method.

A second gravity field model has been obtained through the numerical approach. While the two models show excellent agreement (Figure 1), remaining differences appear to be large on the rim of crater Stickney as this significant topographic anomaly is not represented sufficiently in the coarse analytical model.

### 4. Discussion and Outlook

The gravity fields derived using the analytical and numerical approaches agree to a reasonable level. The harmonic coefficients can readily be used to calculate the gravitational potential on a sphere outside Phobos and can be applied to trajectory analysis of spacecraft near or approaching Phobos.

In contrast, the numerical model will be useful in studying the dynamical environment on the surface of Phobos. Owing to the small size and odd shape of Phobos, orientations of surface acceleration vectors and slope directions cannot be intuitively guessed by

inspection of image data [2]. Centrifugal and tidal forces will partially compensate gravity and must be taken into account. We will investigate the dynamic height as well as surface acceleration on a global scale grid while paying special attention to the proposed region for the 'Phobos-Grunt' landing site. First results will be reported at the meeting.

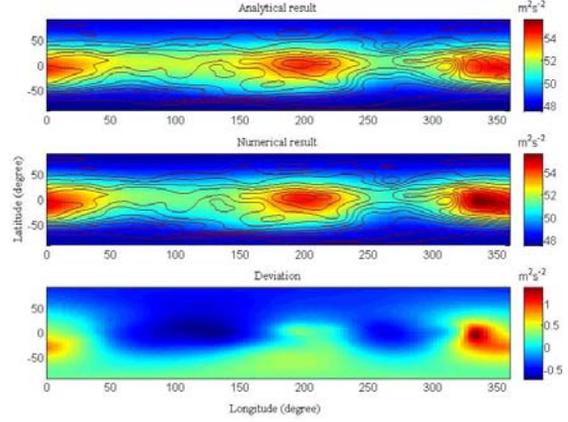


Figure 1: Gravitational potential on a sphere outside Phobos obtained by analytical and numerical methods, and the differences between them. The contours represent the topography.

### References

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