#### **Slab tear propagation:**

#### Numerical model and implications to Apennines



#### Plan of the talk

#### 1. Self-excuses why unpublished (30 min)

#### 2. Talk itself (15 min)



#### **1. Self-excuses**

GERYA

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Gerya Numerical Geodynamic Modelling Second Edition

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#### INTRODUCTION TO Numerical Geodynamic Modelling

Taras Gerya second edition

## Introduction to Numerical Geodynamic Modeling II is out! What's new?



17 chapters b&w figures 67 MatLab examples 21 chapters color figures 90 MatLab examples

## What's new? Free surface stabilization (Chapter 8)

#### 8

The advection equation and marker-in-cell method

**Theory:** Advection equation. Solution methods for continuous and discontinuous variables. Eulerian schemes: upwind differences, flux corrected transport (FCT). Lagrangian schemes: marker-in-cell method. Runge–Kutta advection schemes. Interpolation between markers and nodes. Continuity-based velocity interpolation. 'Sticky air' approach. 'Drunken sailor' instability. Stabilization of free surface.

Exercises: Programming of various advection schemes and markers.



### What's new? Continuity-based marker advection (Chapter 8)

![](_page_6_Figure_1.jpeg)

#### What's new? New 3D elastic stress rotation algorithms (Chapter 12)

According to this approach, 3D rotation is represented by a vorticity pseudo-vector  $(\vec{\omega})$ , which has three components:

12

Elasticity and plasticity

$$\omega_x = \frac{1}{2} \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right), \ \omega_y = \frac{1}{2} \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right), \ \omega_z = \frac{1}{2} \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right).$$
(12.37)

The 3D algorithm can then be summarized as follows (Popov et al., 2014a, personal communication).

(1) Compute the vorticity vector magnitude:

$$\omega_{mag} = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2} . \qquad (12.38)$$

(2) Compute the unit rotation vector  $\vec{n}$ , which also has three components:

$$n_x = \frac{\omega_x}{\omega_{mag}}, \ n_y = \frac{\omega_y}{\omega_{mag}}, \ n_z = \frac{\omega_z}{\omega_{mag}}.$$
 (12.39)

(3) Integrate the incremental rotation angle:

$$=\omega_{mag}\Delta t.$$
 (12.40)

(4) Evaluate the rotation matrix using the Euler-Rodrigues formula:

$$R_{mat} = \cos(\theta) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \sin(\theta) \begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix} + (1 - \cos(\theta)) \begin{pmatrix} n_x n_x & n_x n_y & n_x n_z \\ n_y n_x & n_y n_y & n_y n_z \\ n_z n_x & n_z n_y & n_z n_z \end{pmatrix}.$$
(12.41)

**Theory:** Elastic rheology. Rotation of elastic stresses. Maxwell visco-elastic rheology. Plastic rheology. Plastic yielding criterion. Plastic flow potential. Plastic flow rule. Visco-elasto-plastic rheology.

**Exercises:** Stress buildup/relaxation with a visco-elastic Maxwell rheology, elastic stress rotation programming.

## What's new? Better visco-elasto-plasticity treatment (Chapter 13)

![](_page_8_Figure_1.jpeg)

### What's new? Modeling of compressible materials and inertial processes (Chapter 14)

#### 14

2D thermomechanical modelling of inertial processes

![](_page_9_Figure_3.jpeg)

**Theory:** Numerical implementation of inertia and elastic compressibility. Organization of a thermomechanical code in the case of 2D, visco-elastoplastic deformation with inertia. Thermomechanical iterations. **Exercises:** Programming a 2D thermomechanical code with inertia.

### What's new? Modeling of earthquakes (Chapter 15)

15

Seismo-thermomechanical modelling

![](_page_10_Figure_3.jpeg)

![](_page_10_Figure_4.jpeg)

**Theory:** What is seismo-thermomechanical modelling? Rate-dependent friction. Rate- and state-dependent friction. Regularized rate- and state-dependent friction formulation. Invariant plasticity-like reformulation of rate- and state-dependent friction. Adaptive time stepping. Organization of a seismo-thermomechanical code. Visco-elasto-plastic iterations. **Exercises:** Programming a 2D seismo-thermomechanical code.

![](_page_10_Figure_6.jpeg)

![](_page_10_Figure_7.jpeg)

#### What's new? Modeling of two-phase flow processes (Chapter 16)

![](_page_11_Figure_1.jpeg)

#### **Optimal formulation of conservation laws:**

Using formulation by Yarushina and Podladchikov (2015) based on principles of irreversible thermodynamics

momentum conservation for fluid

🥢 inertia

![](_page_12_Figure_4.jpeg)

mass transfer

mass conservation for fluid

$$\phi\left(\frac{D^f ln\rho^f}{Dt} - \frac{D^s ln\rho^s}{Dt}\right) - \frac{D^s ln(1-\phi)}{Dt} + \operatorname{div}(\vec{q}^D) = \frac{\Gamma^{mass}\rho^t}{\rho^f \rho^s(1-\phi)}$$

momentum conservation for bulk (solid+fluid)

$$\frac{\partial \sigma'_{ij}}{\partial x_j} - \frac{\partial P^t}{\partial x_i} + \rho^t g_i = \left(\rho^f \phi \frac{D^f v_i^f}{Dt}\right) + \left(\rho^s (1 - \phi) \frac{D^s v_i^s}{Dt}\right) + \left(v_i^f - v_i^s\right) \Gamma_{mass}$$

mass conservation for solid

$$\frac{D^{s} ln \rho^{s}}{Dt} + \frac{D^{s} ln(1-\phi)}{Dt} + \operatorname{div}(\vec{v}^{s}) = -\frac{\Gamma^{mass}}{\rho^{s}(1-\phi)}$$

energy conservation for bulk (solid+fluid)

$$(1-\phi)\rho^s C_P^s \frac{D^s T}{Dt} + \phi \rho^f C_P^f \frac{D^f T}{Dt} = -\frac{\partial q_i^t}{\partial x_i} + H_r^t + H_a^t + H_s^t + H_L^t$$

#### **Poro-visco-elasto-plastic rheology:**

Using visco-elasto-plastic formulation by Yarushina and Podladchikov (2015) based on Biot's poroelasticity theory (Biot, 1941)

Maxwell visco-elasto-plastic model:  $\dot{\varepsilon}'_{ij} = \dot{\varepsilon}'_{ij(viscous)} + \dot{\varepsilon}'_{ij(elastic)} + \dot{\varepsilon}'_{ij(plastic)}$ 

where

$$\dot{\varepsilon}_{ij(viscous)}' = \frac{1}{2\eta} \sigma_{ij}',$$

$$\dot{\varepsilon}_{ij(elastic)}' = \frac{1}{2\mu} \frac{D \sigma_{ij}'}{Dt},$$

$$\begin{split} \dot{\varepsilon}_{ij(plastic)}^{'} &= 0 \text{ for } \sigma_{\mathrm{II}} < \sigma_{\mathrm{yield}}, \\ \dot{\varepsilon}_{ij(plastic)}^{'} &= \chi \frac{\partial G_{plastic}}{\partial \sigma_{ij}^{'}} = \chi \frac{\sigma_{ij}^{'}}{2\sigma_{\mathrm{II}}} \text{ for } \sigma_{\mathrm{II}} = \sigma_{yield}, \\ G_{plastic} &= \sigma_{\mathrm{II}}, \end{split}$$

$$\sigma_{\rm II} = \sqrt{\frac{1}{2}{\sigma'_{ij}}^2},$$

Drucker-Prager plasticity model with fluid pressure:  $\sigma_{II} = \sigma_{yield}$ 

critical hydro-mechanical feedback  

$$\sigma_{yield} = \sigma_c + \gamma_{int} (P^t - P^f)$$
 when  $P^t - P^f > \frac{\sigma_c - \sigma_t}{1 - \gamma_{int}}$  (confined fractures)  
 $\sigma_{yield} = \sigma_t + (P^t - P^f)$  when  $P^t - P^f < \frac{\sigma_c - \sigma_t}{1 - \gamma_{int}}$  (tensile fractures)  
 $P^t - (1 - \phi)P^s + \phi P^f$ 

![](_page_13_Picture_10.jpeg)

#### Self-consistent formulation of closure relations:

Using visco-elasto-plastic formulation by Yarushina and Podladchikov (2015) based on Biot's poroelasticity theory (Biot, 1941)

![](_page_14_Picture_2.jpeg)

 $\frac{D^{s}ln(1-\phi)}{Dt} = \frac{\beta^{\phi}}{(1-\phi)} \left( \frac{D^{s}P^{t}}{Dt} - \frac{D^{f}P^{f}}{Dt} \right) - \frac{\alpha^{\phi}}{(1-\phi)} \frac{D^{s}T}{Dt} + \frac{P^{t} - P^{f}}{(1-\phi)\eta^{\phi}} - \underbrace{\Gamma^{mass}A^{\phi}}_{T},$  $\frac{D^{s}ln\rho^{s}}{Dt} = \frac{\beta^{s}}{1-\phi} \left( \frac{D^{s}P^{t}}{Dt} - \phi \frac{D^{f}P^{f}}{Dt} \right) - \alpha^{s} \frac{D^{s}T}{Dt} + \underbrace{\Gamma^{mass}A^{s}}_{T},$  $\frac{D^{f}ln\rho^{f}}{Dt} = \beta^{f} \frac{D^{f}P^{f}}{Dt} - \alpha^{f} \frac{D^{f}T}{Dt} + \underbrace{\Gamma^{mass}A^{f}}_{T},$ 

#### How to formulate mass transfer related terms? C-component approach (C-component = Chemical trash can)

![](_page_15_Picture_1.jpeg)

- We characterize complex reactive mass transfer by considering a net mass transfer ΔM=Σ ΔMi, where ΔMi is the mass of i-th chemical component transferred from the solid to the fluid during a time increment Δt: positive ΔMi values corresponds to the mass transfer from the solid to the fluid (dehydration, melting, dissolution, etc.), whereas negative ΔMi values imply the mass transfer from the fluid to the solid (hydration, solidification, precipitation, etc.).
- The transferred mass ΔM is formally described as a single chemically complex pseudo-component C of the solid and fluid. Stoichiometry of C-component is given by C=ΣΔMi/ΔM\*Ci, where Ci is chemical formula of i-th chemical component.
- 3. C-component has different density in its solid ( $\rho_c^s$ ) and fluid ( $\rho_c^f$ ) state, which can also differ from the bulk density of the solid ( $\rho^s$ ) and fluid ( $\rho^f$ ).

![](_page_15_Figure_5.jpeg)

# How to formulate mass transfer related terms with C-component approach?

![](_page_16_Picture_1.jpeg)

mass conservation for solid (porous matrix divergence depends on mass transfer)

$$\operatorname{div}(\vec{v}^{s}) + \beta_{d} \left( \frac{D^{s}P^{t}}{Dt} - K_{BW} \frac{D^{f}P^{f}}{Dt} \right) + \frac{P^{t} - P^{f}}{(1 - \phi)\eta^{\phi}} = \left( \alpha^{s} + \frac{\alpha^{\phi}}{(1 - \phi)} \right) \frac{D^{s}T}{Dt} + \Gamma_{mass} \left( \frac{1}{\rho_{C}^{f}} - \frac{1}{\rho_{C}^{s}} \right)$$

mass conservation for fluid (Darcy flux divergence is independent of mass transfer)  $\operatorname{div}(\vec{q}^{D}) - K_{BW}\beta_d \left(\frac{D^s P^t}{Dt} - \frac{1}{K_{Sk}}\frac{D^f P^f}{Dt}\right) - \frac{P^t - P^f}{(1 - \phi)\eta^{\phi}} = \phi \left[\alpha^f \frac{D^f T}{Dt} - \left(\alpha^s + \frac{\alpha^{\phi}}{(1 - \phi)\phi}\right)\frac{D^s T}{Dt}\right]$ 

The advantage of C-component approach is that the form of the discretized conservation equations becomes independent of the actual chemistry, thermodynamics and kinetics of mass transfer, which can be computed separately during SHTMC-iteration.

#### How to formulate mass transfer related terms with C-component approach

![](_page_17_Picture_1.jpeg)

mass conservation for solid (porous matrix divergence depends on mass transfer)

$$\operatorname{div}(\vec{v}^s) + \beta_d \left( \frac{D^s P^t}{Dt} - K_{BW} \frac{D^f P^f}{Dt} \right) + \frac{P^t - P^f}{(1 - \phi)\eta^{\phi}} = \left( \alpha^s + \frac{\alpha^{\phi}}{(1 - \phi)} \right) \frac{D^s T}{Dt} + \Gamma_{mass} \left( \frac{1}{\rho_C^f} - \frac{1}{\rho_C^s} \right)$$

 $\Gamma_{mass}, A^{\phi}, A^{s}$  and  $A^{f}$  can be formulated locally as a function of *six independent* quantities: porosity and density of the solid and fluid before (i.e., for the beginning of the time step,  $\phi^{o}, \rho^{so}, \rho^{fo}$ ) and after (i.e., for the end of the time step,  $\phi, \rho^{s}, \rho^{f}$ ) chemical reactions.

$$\Gamma_{mass}\left(\frac{1}{\rho_C^f} - \frac{1}{\rho_C^s}\right) = \frac{1 - R_V}{\Delta t}$$
INITIAL/FINAL volume ratio
$$R_V = \frac{V^o}{V} = \frac{\rho^s(1 - \phi) + \rho^f \phi}{\rho^{so}(1 - \phi^o) + \rho^{fo} \phi^o}$$
FINAL/INITIAL density ratio

#### How to formulate mass transfer related terms with C-component approach

![](_page_18_Picture_1.jpeg)

INITIAL/FINAL volume ratio 
$$R_V = \frac{V^o}{V} = \frac{\rho^s(1-\phi) + \rho^f \phi}{\rho^{so}(1-\phi^o) + \rho^{fo} \phi^o}$$

FINAL/INITIAL density ratio

$$\Gamma^{mass} = \frac{1}{V} \frac{\Delta M}{\Delta t} = \frac{R_V \rho^{so} (1 - \phi^o) - \rho^s (1 - \phi)}{\Delta t} = \frac{\rho^f \phi - R_V \rho^{fo} \phi^o}{\Delta t},$$

$$\Gamma^{mass}A^{\phi} = \frac{R_V(\phi - \phi^o)}{\Delta t(1 - \phi)}$$

$$\Gamma^{mass}A^s = \frac{R_V}{\Delta t} \left(\frac{1-\phi^o}{1-\phi}\right) \left(1-\frac{\rho^{so}}{\rho^s}\right)$$

$$\Gamma^{mass}A^{f} = \frac{R_{V}\phi^{o}}{\Delta t\phi} \left(1 - \frac{\rho^{fo}}{\rho^{f}}\right)$$

#### How to implement coupled SHTMC equations? Global iteration

![](_page_19_Picture_1.jpeg)

![](_page_19_Figure_2.jpeg)

## **Examples**

![](_page_20_Picture_1.jpeg)

![](_page_20_Figure_2.jpeg)

## What's new? Adaptive mesh refinement (Chapter 17)

![](_page_21_Figure_1.jpeg)

Theory: What is AMR? Mesh refinement with staggered finite differences. 'Swiss cross' approach. Block-structured AMR approach. Conditions for 'hanging nodes'. Refinement criteria. Convergence of the numerical solution. AMR for different conservation equations. Exercises: Programming of AMR code.

#### What's new? Color figures, 90 MatLab examples.

![](_page_22_Figure_1.jpeg)

1.345 Myr

2.001 Myr

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#### **Slab tear propagation:**

#### Numerical model and implications to Apennines

![](_page_24_Picture_2.jpeg)

**Modeler's classification of geodynamical processes** 

Easy to imagine and Easy to model

Difficult to imagine but Easy to model

#### Difficult to imagine and Difficult to model

![](_page_25_Picture_4.jpeg)

#### Wortel and Spakman (2000)

![](_page_26_Figure_1.jpeg)

Fig. 4. Plate boundary processes predicted to accompany lateral migration of slab detachment. The concentration of slab pull forces causes a pattern of subsidence (depocenter development) and uplift migrating along strike. It also enhances arc migration (roll-back). Asthenospheric material flows into the gap resulting from slab detachment and causes a specific type of variable composition magmatism, of finite duration, and possibly mineralization.

#### Gerya et al. (2004)

![](_page_26_Figure_4.jpeg)

Fig. 8. Conceptual 3-D model of the slab breakoff geometry inferred from our 2-D numerical experiments. See the text for discussion.

#### Easy to imagine but Difficult to model

## Motivation

# **3D slab tearing under Appenines**

C. Faccenna et al. / Earth and Planetary Science Letters 407 (2014) 163-174

![](_page_28_Figure_2.jpeg)

![](_page_29_Figure_0.jpeg)

Fig. 5. 3D view of the topography on top of the isosurface enclosing the >0.8% anomaly volume from *P* wave M01 tomography model (Piromallo and Morelli, 2003), where the colors indicate depth in km. Notice the correspondence between low topography and locations of active subduction with attached slab segments and the slab window beneath the Central-Southern Apennines. Modified from Faccenna et al. (2011).

Faccenna et al. (2014)

![](_page_30_Picture_0.jpeg)

![](_page_31_Figure_0.jpeg)

![](_page_32_Figure_0.jpeg)

![](_page_33_Figure_0.jpeg)

![](_page_34_Figure_0.jpeg)

Previous 3D models (advancing subduction)




Numerical Model (retreating subduction)























































 $v_{tear} = v_{subduction} / \cos(\alpha)$ if  $\alpha = 90^{\circ}$  then  $v_{tear} = \infty$ 

Short initial weak zones
















## Conclusions

- **1. Retreating subduction is very efficient in causing slab tearing**
- 2. Lateral tear propagation is controlled by the passive margin obliquity:  $v_{tear} = v_{subduction} / \cos(\alpha)$ , where  $\alpha$  is the angle between the margin and subduction direction
- 3. Subduction may continue with different polarity after tearing

