Generation and transport of liquid water in the high-pressure ice layers of Ganymede and Titan

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of Mantle and Lithosphere Dynamics,
La Certosa di Pontignano, Italy,
August 26, 2019


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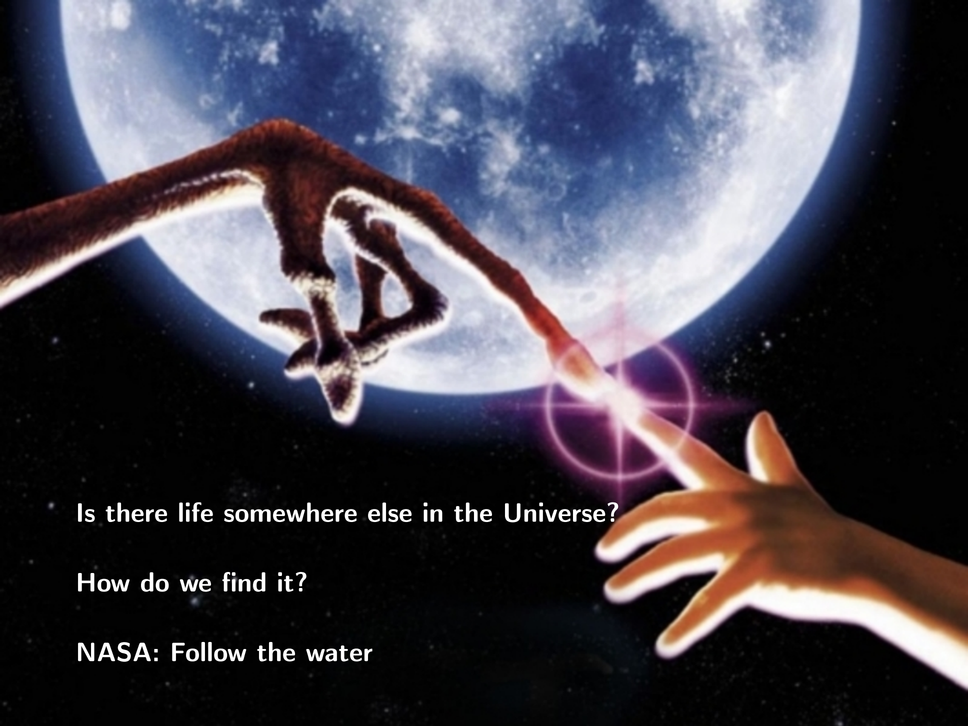
How do we find it?



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NASA: Follow the water

Follow the water

- life might have started at the bottom of Earth's oceans


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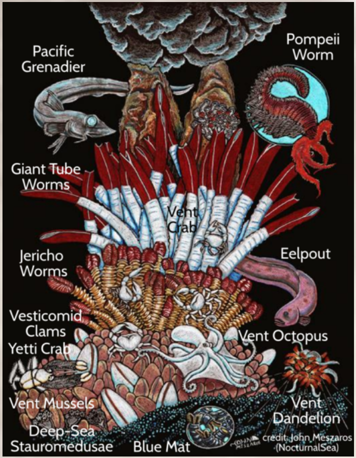

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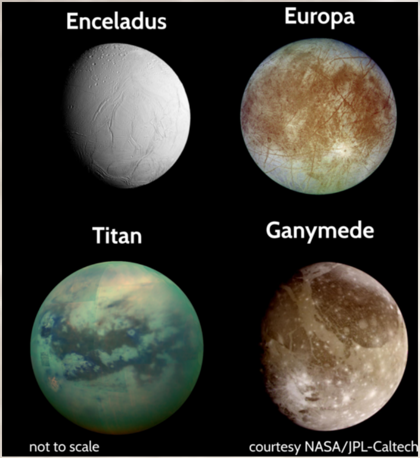

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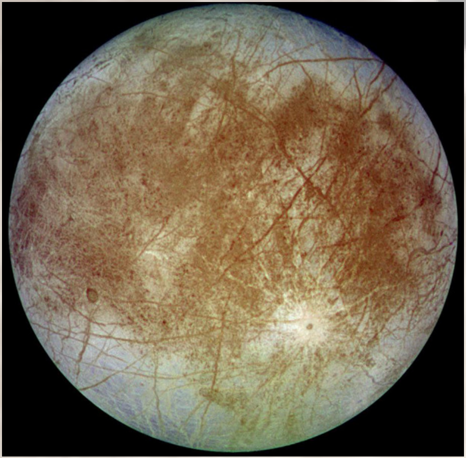

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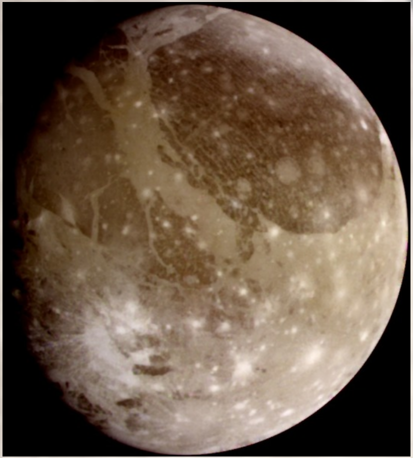

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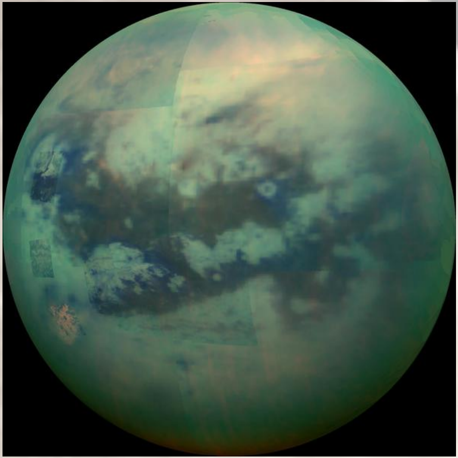

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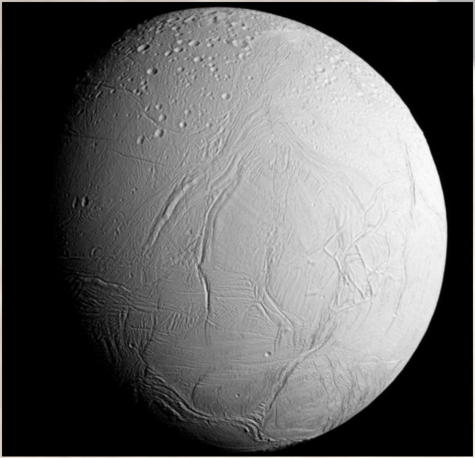
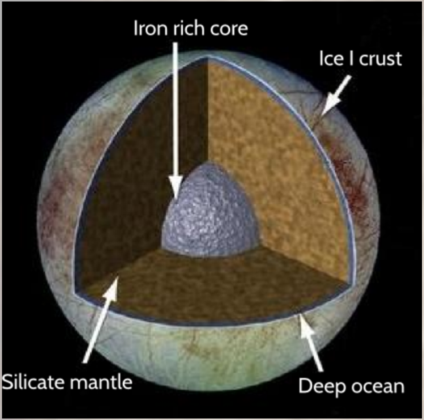
- Enceladus:
  - geysers
  - libration of outer layer



Europa & Enceladus

- direct contact of silicates with the ocean
- conditions similar to the Earth oceans - emergence of life?



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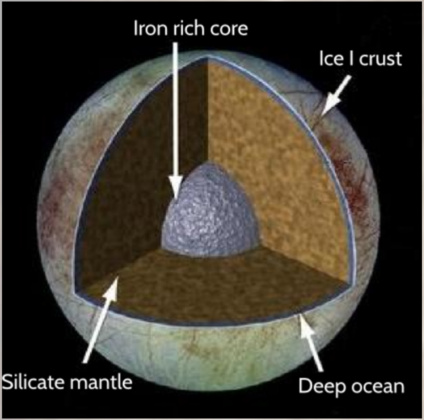
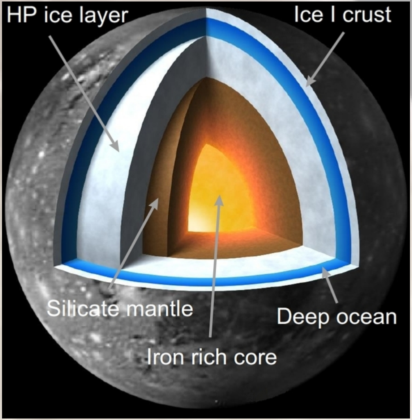


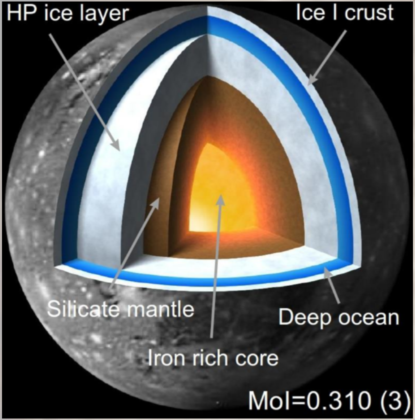
Ganymede & Titan

- thick HP ice layer between silicates and the ocean
- is water & material exchange possible?



Ganymede

- full differentiation: MoI=0.310
- tenuous O_2 exosphere
- intrinsic magnetic field



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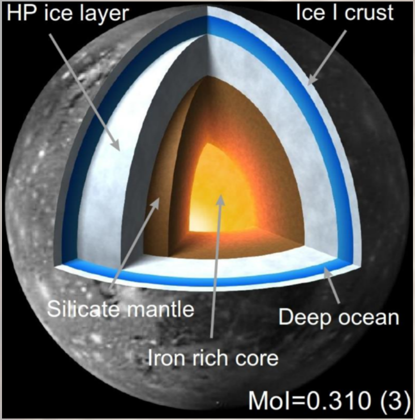
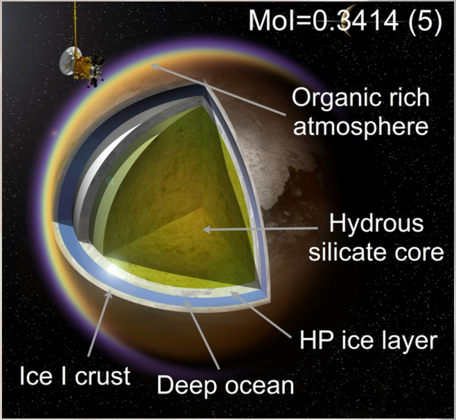


Titan

- partial differentiation: MoI=0.3414
- dense N_2-CH_4-Ar atmosphere

→ possibly ongoing exchange?



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- dehydrated mantle
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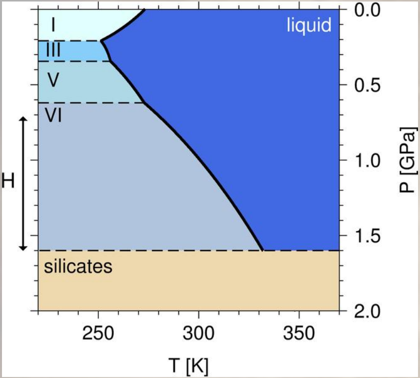
- $H \gtrsim 400$ km: only ice VI



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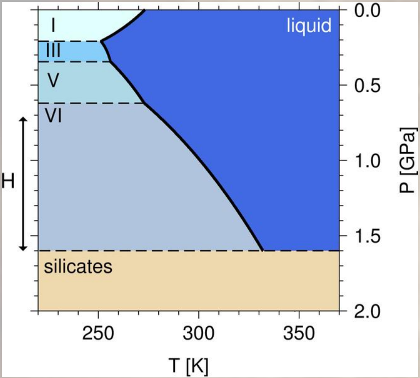
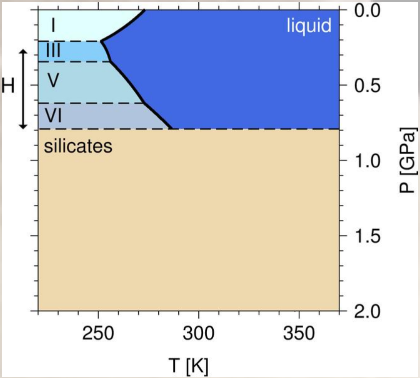


Titan

- core of hydrated silicates
  + thinner H_2O layer
- $H \sim 50$–300 km: ices VI, V, III?



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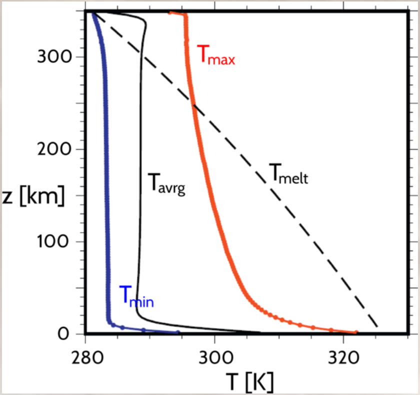
- effect of ice phases transitions on convection:

a) linear stability analysis (*Bercovici +*, 1986; *Sotin & Parmentier*,1989)
b) 2d convection in ice VI and VII mantle (*Journaux & Noack*, 2015)

Heat and material exchange

- *Choblet +* (2017): solid-state thermal convection in 3d spherical
- T_{max} and $T_{\mathrm{avrg}} > T_{\mathrm{melt}}$ in the upper part



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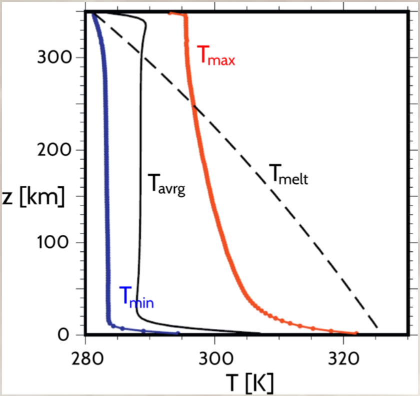

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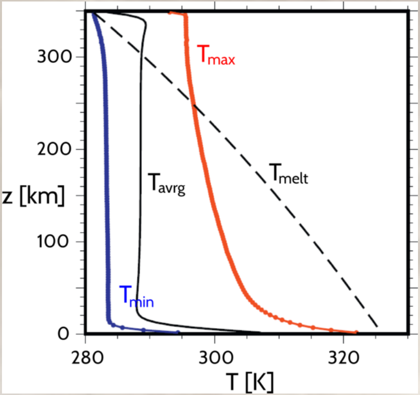
$\rightarrow$ **two-phase mixture model** (*Bercovici +*, 2001; *Šrámek +*, 2007)



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- different material densities $\rightarrow$ 2 mass balances
- phase separation $\rightarrow$ 2 linear momentum balances
- thermal equilibrium between the two phases $\rightarrow$ 1 energy balance

Governing equations

1. Thermal convection in ice - incompressible extended Boussinesq approximation

- mass balance of ice

$$\nabla \cdot \mathbf{v}_\mathrm{i} = 0$$

- linear momentum balance of ice

$$0 = -\nabla \Pi - \rho_\mathrm{i} \alpha_\mathrm{i} T \boldsymbol{g} + \nabla \cdot \boldsymbol{\sigma}_\mathrm{i}$$

- energy balance of ice

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \mathbf{v}_\mathrm{i} \cdot \nabla T - \rho_\mathrm{i} T \alpha_\mathrm{i} \mathbf{v}_\mathrm{i} \cdot \boldsymbol{g} = \nabla \cdot (k_\mathrm{i} \nabla T) + \boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i}$$

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3. $\mu_\mathrm{w} \ll \mu_\mathrm{i}$: $\overline{\boldsymbol{\sigma}} \rightarrow (1-\phi)\boldsymbol{\sigma}_\mathrm{i}$, $T_\mathrm{w} = T^\mathrm{m}$: $\overline{\rho\alpha} \rightarrow (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i}$, $\overline{\rho c^\mathrm{p}} \rightarrow (1-\phi)\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \rightarrow \rho_\mathrm{i} c_\mathrm{i}^\mathrm{p}$

- mass balance of mixture

$$\nabla \cdot \bar{\mathbf{v}} = 0$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T\boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i}$$

- energy balance of mixture

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p} \boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\bar{k} \nabla T) + (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i}$$

Governing equations

1. Thermal convection in ice - incompressible extended Boussinesq approximation
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3. $\mu_{\mathrm{w}} \ll \mu_{\mathrm{i}}$: $\overline{\boldsymbol{\sigma}} \rightarrow (1-\phi)\boldsymbol{\sigma}_{\mathrm{i}}$, $T_{\mathrm{w}} = T^{\mathrm{m}}$: $\overline{\rho\alpha} \rightarrow (1-\phi)\rho_{\mathrm{i}}\alpha_{\mathrm{i}}$, $\overline{\rho c^{\mathrm{p}}} \rightarrow (1-\phi)\rho_{\mathrm{i}} c_{\mathrm{i}}^{\mathrm{p}} \rightarrow \rho_{\mathrm{i}} c_{\mathrm{i}}^{\mathrm{p}}$
4. Melting/freezing

- mass balance of mixture

$$\nabla \cdot \overline{\mathbf{v}} = 0$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_{\mathrm{i}}\alpha_{\mathrm{i}} T\boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_{\mathrm{i}}$$

- energy balance of mixture

$$\rho_{\mathrm{i}} c_{\mathrm{i}}^{\mathrm{p}} \frac{\partial T}{\partial t} + \overline{\rho c^{\mathrm{p}} \boldsymbol{v}} \cdot \nabla T - \rho_{\mathrm{i}} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\overline{k} \nabla T) + (1-\phi)\boldsymbol{\sigma}_{\mathrm{i}} : \nabla \mathbf{v}_{\mathrm{i}}$$

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4. Melting/freezing $\rightarrow$ compressible mixture

- mass balance of mixture

$$\nabla \cdot \overline{\mathbf{v}} = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}} \Gamma_\mathrm{m}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T \boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i}$$

- energy balance of mixture

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p} \boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\overline{k} \nabla T) + (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i}$$

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4. Melting/freezing $\to$ compressible mixture

- mass balance of mixture

$$\nabla \cdot \mathbf{v}_\mathrm{i} + \nabla \cdot [\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}}\Gamma_\mathrm{m}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T\boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i}$$

- energy balance of mixture

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p}\frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p}\boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T\overline{\alpha\boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\bar{k}\nabla T) + (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla\mathbf{v}_\mathrm{i}$$

Governing equations

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4. Melting/freezing $\to$ compressible mixture $\to$ $\nabla \cdot \mathbf{v}_{\rm i} \neq 0$

- mass balance of mixture

$$\nabla \cdot \mathbf{v}_{\rm i} + \nabla \cdot [\phi(\mathbf{v}_{\rm w} - \mathbf{v}_{\rm i})] = \frac{\Delta\rho}{\rho_{\rm i}\rho_{\rm w}}\Gamma_{\rm m}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_{\rm i}\alpha_{\rm i} T\boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_{\rm i}$$

- energy balance of mixture

$$\rho_{\rm i} c_{\rm i}^{\rm p} \frac{\partial T}{\partial t} + \overline{\rho c^{\rm p} \boldsymbol{v}} \cdot \nabla T - \rho_{\rm i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\overline{k} \nabla T) + (1-\phi)\boldsymbol{\sigma}_{\rm i} : \nabla \mathbf{v}_{\rm i}$$

Governing equations

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4. Melting/freezing $\rightarrow$ compressible mixture $\rightarrow \nabla \cdot \mathbf{v}_{\mathrm{i}} \neq 0 \rightarrow \zeta_{\mathrm{i}} \sim \frac{\mu_{\mathrm{i}}}{\phi}$

- mass balance of mixture

$$\nabla \cdot \mathbf{v}_{\mathrm{i}} + \nabla \cdot [\phi(\mathbf{v}_{\mathrm{w}} - \mathbf{v}_{\mathrm{i}})] = \frac{\Delta\rho}{\rho_{\mathrm{i}}\rho_{\mathrm{w}}}\Gamma_{\mathrm{m}}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_{\mathrm{i}}\alpha_{\mathrm{i}} T\boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_{\mathrm{i}}$$

- energy balance of mixture

$$\rho_{\mathrm{i}} c_{\mathrm{i}}^{\mathrm{p}} \frac{\partial T}{\partial t} + \overline{\rho c^{\mathrm{p}} \boldsymbol{v}} \cdot \nabla T - \rho_{\mathrm{i}} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\overline{k} \nabla T) + (1-\phi)\boldsymbol{\sigma}_{\mathrm{i}} : \nabla \mathbf{v}_{\mathrm{i}}$$

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4. Melting/freezing $\to$ compressible mixture $\to \nabla \cdot \mathbf{v}_\mathrm{i} \neq 0 \to \zeta_\mathrm{i} \sim \frac{\mu_\mathrm{i}}{\phi}$

- mass balance of mixture

$$\nabla \cdot \mathbf{v}_\mathrm{i} + \nabla \cdot [\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}} \Gamma_\mathrm{m}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T \boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i} + \nabla \left[\frac{(1-\phi)\mu_\mathrm{i}}{\phi} (\nabla \cdot \mathbf{v}_\mathrm{i}) \right]$$

- energy balance of mixture

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p} \boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\bar{k} \nabla T) + (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i}$$

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- mass balance of mixture

$$\nabla \cdot \mathbf{v}_\mathrm{i} + \nabla \cdot [\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}} \Gamma_\mathrm{m}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T\boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i} + \nabla \left[\frac{(1-\phi)\mu_\mathrm{i}}{\phi} (\nabla \cdot \mathbf{v}_\mathrm{i}) \right]$$

- energy balance of mixture

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p} \boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\bar{k} \nabla T) + (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i} + \frac{1-\phi}{\phi} \mu_\mathrm{i} (\nabla \cdot \mathbf{v}_\mathrm{i})^2$$

Governing equations

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- mass balance of mixture

$$\nabla \cdot \mathbf{v}_\mathrm{i} + \nabla \cdot [\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}} \Gamma_\mathrm{m}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T \boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i} + \nabla \left[\frac{(1-\phi)\mu_\mathrm{i}}{\phi} (\nabla \cdot \mathbf{v}_\mathrm{i}) \right]$$

- energy balance of mixture ($\tilde{\Psi} = (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i} + \frac{1-\phi}{\phi} \mu_\mathrm{i} (\nabla \cdot \mathbf{v}_\mathrm{i})^2$)

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p} \boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\bar{k} \nabla T) + \tilde{\Psi}$$

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4. Melting/freezing $\rightarrow$ compressible mixture $\rightarrow \nabla \cdot \mathbf{v}_\mathrm{i} \neq 0 \rightarrow \zeta_\mathrm{i} \sim \frac{\mu_\mathrm{i}}{\phi}$

   $\rightarrow$ latent heat L_m consumption/release

- mass balance of mixture

$$\nabla \cdot \mathbf{v}_\mathrm{i} + \nabla \cdot [\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}} \Gamma_\mathrm{m}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T \boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i} + \nabla \left[\frac{(1-\phi)\mu_\mathrm{i}}{\phi} (\nabla \cdot \mathbf{v}_\mathrm{i}) \right]$$

- energy balance of mixture ($\tilde{\Psi} = (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i} + \frac{1-\phi}{\phi} \mu_\mathrm{i} (\nabla \cdot \mathbf{v}_\mathrm{i})^2$)

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p} \boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} = \nabla \cdot (\overline{k} \nabla T) + \tilde{\Psi}$$

Governing equations

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4. Melting/freezing $\to$ compressible mixture $\to \nabla \cdot \mathbf{v}_{\mathrm{i}} \neq 0 \to \zeta_{\mathrm{i}} \sim \frac{\mu_{\mathrm{i}}}{\phi}$
   $\to$ latent heat L_{m} consumption/release

- mass balance of mixture

$$\nabla \cdot \mathbf{v}_{\mathrm{i}} + \nabla \cdot [\phi(\mathbf{v}_{\mathrm{w}} - \mathbf{v}_{\mathrm{i}})] = \frac{\Delta\rho}{\rho_{\mathrm{i}}\rho_{\mathrm{w}}} \Gamma_{\mathrm{m}}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_{\mathrm{i}}\alpha_{\mathrm{i}} T \boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_{\mathrm{i}} + \nabla \left[\frac{(1-\phi)\mu_{\mathrm{i}}}{\phi} (\nabla \cdot \mathbf{v}_{\mathrm{i}}) \right]$$

- energy balance of mixture ($\tilde{\Psi} = (1-\phi)\boldsymbol{\sigma}_{\mathrm{i}} : \nabla \mathbf{v}_{\mathrm{i}} + \frac{1-\phi}{\phi} \mu_{\mathrm{i}} (\nabla \cdot \mathbf{v}_{\mathrm{i}})^2$)

$$\rho_{\mathrm{i}} c_{\mathrm{i}}^{\mathrm{p}} \frac{\partial T}{\partial t} + \overline{\rho c^{\mathrm{p}} \boldsymbol{v}} \cdot \nabla T - \rho_{\mathrm{i}} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} + \Gamma_{\mathrm{m}} L_{\mathrm{m}} = \nabla \cdot (\overline{k} \nabla T) + \tilde{\Psi}$$

Governing equations

1. Thermal convection in ice - incompressible extended Boussinesq approximation
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4. Melting/freezing $\to$ compressible mixture $\to \nabla \cdot \mathbf{v}_\mathrm{i} \neq 0 \to \zeta_\mathrm{i} \sim \frac{\mu_\mathrm{i}}{\phi}$
   $\to$ latent heat L_m consumption/release ; solid phase change

- mass balance of mixture

$$\nabla \cdot \mathbf{v}_\mathrm{i} + \nabla \cdot [\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}} \Gamma_\mathrm{m}$$

- linear momentum balance of ice

$$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T \boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i} + \nabla \left[\frac{(1-\phi)\mu_\mathrm{i}}{\phi} (\nabla \cdot \mathbf{v}_\mathrm{i}) \right]$$

- energy balance of mixture ($\tilde{\Psi} = (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i} + \frac{1-\phi}{\phi} \mu_\mathrm{i} (\nabla \cdot \mathbf{v}_\mathrm{i})^2$)

$$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p} \boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} + \Gamma_\mathrm{m} L_\mathrm{m} + \Gamma_\mathrm{s} L_\mathrm{s} = \nabla \cdot (\bar{k} \nabla T) + \tilde{\Psi}$$

Governing equations

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4. Melting/freezing $\to$ compressible mixture $\to \nabla \cdot \mathbf{v}_\mathrm{i} \neq 0 \to \zeta_\mathrm{i} \sim \frac{\mu_\mathrm{i}}{\phi}$
   $\to$ latent heat L_m consumption/release ; solid phase change
5. Melt buoyancy: $\Delta\rho = \rho_\mathrm{i} - \rho_\mathrm{w} \neq 0$
   - mass balance of mixture
   $$\nabla \cdot \mathbf{v}_\mathrm{i} + \nabla \cdot [\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}} \Gamma_\mathrm{m}$$
   - linear momentum balance of ice
   $$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T \boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i} + \nabla \left[\frac{(1-\phi)\mu_\mathrm{i}}{\phi} (\nabla \cdot \mathbf{v}_\mathrm{i}) \right]$$
   - energy balance of mixture ($\tilde{\Psi} = (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla \mathbf{v}_\mathrm{i} + \frac{1-\phi}{\phi} \mu_\mathrm{i} (\nabla \cdot \mathbf{v}_\mathrm{i})^2$)
   $$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p} \frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p} \boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T \overline{\alpha \boldsymbol{v}} \cdot \boldsymbol{g} + \Gamma_\mathrm{m} L_\mathrm{m} + \Gamma_\mathrm{s} L_\mathrm{s} = \nabla \cdot (\bar{k} \nabla T) + \tilde{\Psi}$$

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4. Melting/freezing $\rightarrow$ compressible mixture $\rightarrow \nabla \cdot \mathbf{v}_\mathrm{i} \neq 0 \rightarrow \zeta_\mathrm{i} \sim \frac{\mu_\mathrm{i}}{\phi}$
   $\rightarrow$ latent heat L_m consumption/release ; solid phase change
5. Melt buoyancy: $\Delta\rho = \rho_\mathrm{i} - \rho_\mathrm{w} \neq 0$
   - mass balance of mixture
     $$\nabla \cdot \mathbf{v}_\mathrm{i} + \nabla \cdot [\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}}\Gamma_\mathrm{m}$$
   - linear momentum balance of ice
     $$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T\boldsymbol{g} - \phi\Delta\rho\boldsymbol{g} + \nabla \cdot (1-\phi)\boldsymbol{\sigma}_\mathrm{i} + \nabla\left[\frac{(1-\phi)\mu_\mathrm{i}}{\phi}(\nabla \cdot \mathbf{v}_\mathrm{i})\right]$$
   - energy balance of mixture ($\tilde{\Psi} = (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla\mathbf{v}_\mathrm{i} + \frac{1-\phi}{\phi}\mu_\mathrm{i}(\nabla \cdot \mathbf{v}_\mathrm{i})^2$)
     $$\rho_\mathrm{i} c_\mathrm{i}^\mathrm{p}\frac{\partial T}{\partial t} + \overline{\rho c^\mathrm{p}\boldsymbol{v}} \cdot \nabla T - \rho_\mathrm{i} T\overline{\alpha\boldsymbol{v}} \cdot \boldsymbol{g} + \Gamma_\mathrm{m} L_\mathrm{m} + \Gamma_\mathrm{s} L_\mathrm{s} = \nabla \cdot (\overline{k}\nabla T) + \tilde{\Psi}$$

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4. Melting/freezing $\rightarrow$ compressible mixture $\rightarrow \nabla\cdot\mathbf{v}_\mathrm{i} \neq 0 \rightarrow \zeta_\mathrm{i} \sim \frac{\mu_\mathrm{i}}{\phi}$
   $\rightarrow$ latent heat L_m consumption/release ; solid phase change
5. Melt buoyancy: $\Delta\rho = \rho_\mathrm{i} - \rho_\mathrm{w} \neq 0 \rightarrow$ water percolation through ice
   - mass balance of mixture

     $$\nabla\cdot\mathbf{v}_\mathrm{i} + \nabla\cdot[\phi(\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i})] = \frac{\Delta\rho}{\rho_\mathrm{i}\rho_\mathrm{w}}\Gamma_\mathrm{m}$$

   - linear momentum balance of ice

     $$0 = -\nabla\Pi - (1-\phi)\rho_\mathrm{i}\alpha_\mathrm{i} T\boldsymbol{g} - \phi\Delta\rho\boldsymbol{g} + \nabla\cdot(1-\phi)\boldsymbol{\sigma}_\mathrm{i} + \nabla\left[\frac{(1-\phi)\mu_\mathrm{i}}{\phi}(\nabla\cdot\mathbf{v}_\mathrm{i})\right]$$

   - energy balance of mixture ($\tilde{\Psi} = (1-\phi)\boldsymbol{\sigma}_\mathrm{i} : \nabla\mathbf{v}_\mathrm{i} + \frac{1-\phi}{\phi}\mu_\mathrm{i}(\nabla\cdot\mathbf{v}_\mathrm{i})^2$)

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     $$\mathbf{v}_\mathrm{w} - \mathbf{v}_\mathrm{i} = -\frac{K_\phi}{\mu_\mathrm{w}\phi}(1-\phi)\Delta\rho\boldsymbol{g}$$

Governing equations

1. Thermal convection in ice - incompressible extended Boussinesq approximation
2. Mixture of two incompressible components: $\xi_\mathrm{i} \to \overline{\xi} := (1-\phi)\xi_\mathrm{i} + \phi\xi_\mathrm{w}$
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4. Melting/freezing $\to$ compressible mixture $\to \nabla\cdot\mathbf{v}_\mathrm{i} \neq 0 \to \zeta_\mathrm{i} \sim \frac{\mu_\mathrm{i}}{\phi}$
   $\to$ latent heat L_m consumption/release ; solid phase change
5. Melt buoyancy: $\Delta\rho = \rho_\mathrm{i} - \rho_\mathrm{w} \neq 0 \to$ water percolation through ice
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- mass balance of water $\to$ advection of porosity

$$\frac{\partial\phi}{\partial t} + \nabla \cdot (\phi\mathbf{v}_\mathrm{w}) = \frac{\Gamma_\mathrm{m}}{\rho_\mathrm{w}}$$

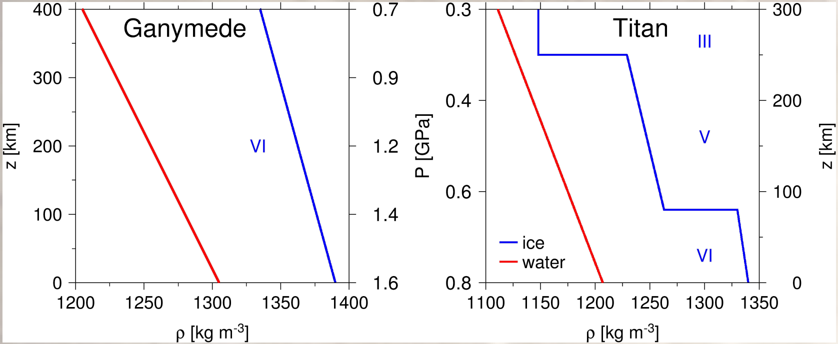
Material parameters

- ice permeable to water transport above a threshold porosity ϕ_c

$$K_\phi = \begin{cases} k_0(\phi - \phi_c)^2 & \phi \geq \phi_c \\ 0 & \phi < \phi_c \end{cases}$$

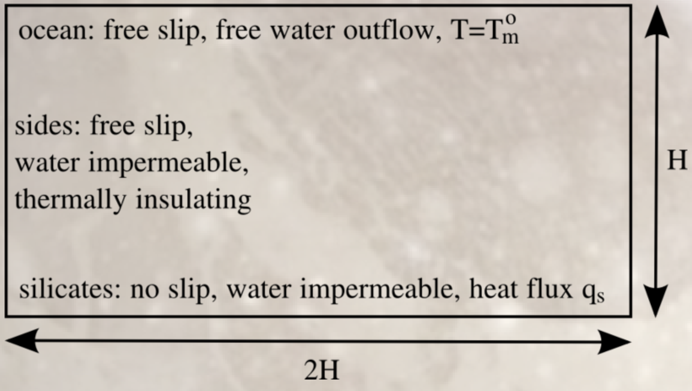
- temperature and depth dependent ice viscosity

$$\mu_i = \mu_0 \exp\left[\frac{Q}{R}\left(\frac{1}{T} - \frac{1}{T^m(z)}\right)\right]$$



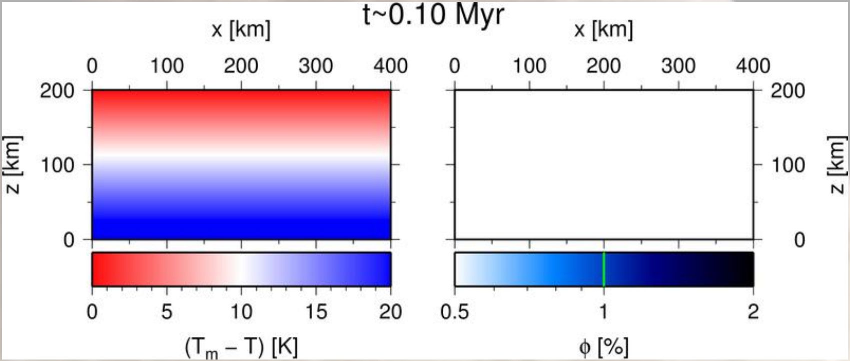
Implementation

- FEniCS (fenicsproject.org, *Logg +*, 2012; *Alnaes +*, 2015), MPI



Reference simulation for Ganymede

H=200 km, μ_i=10^{15} Pa s, q_s=20 mW m^{-2}



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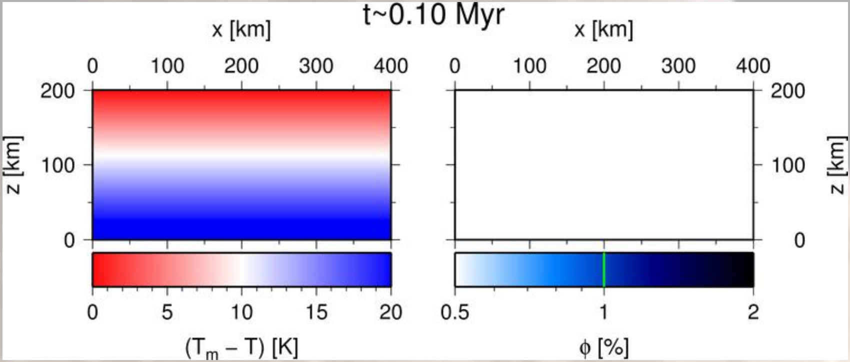

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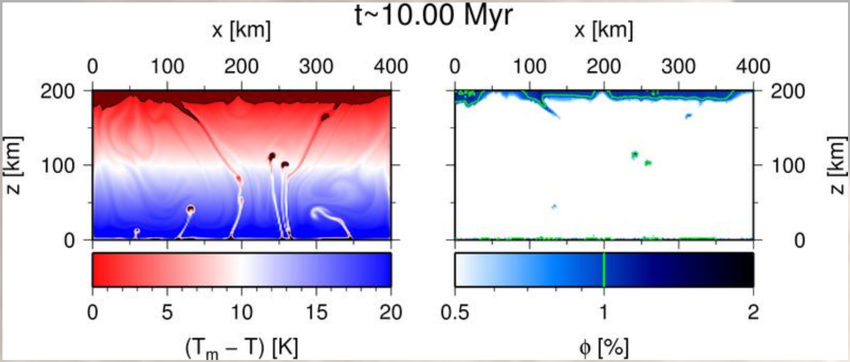

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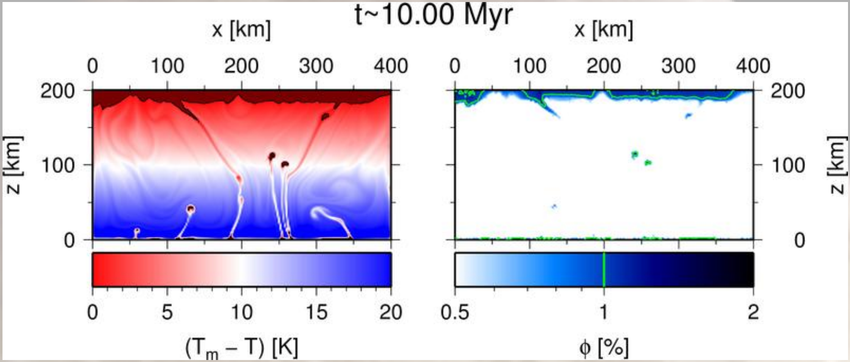

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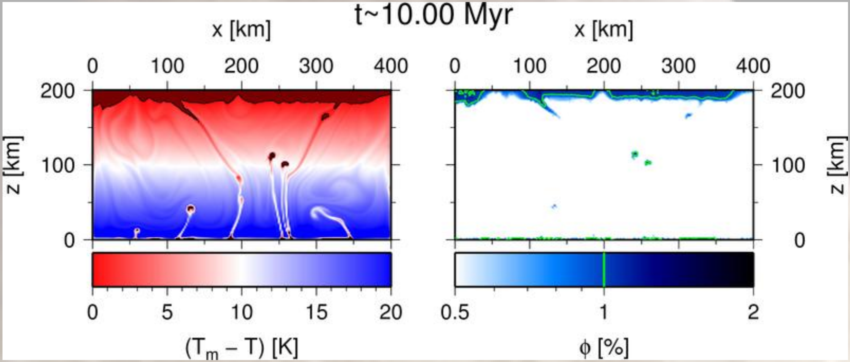
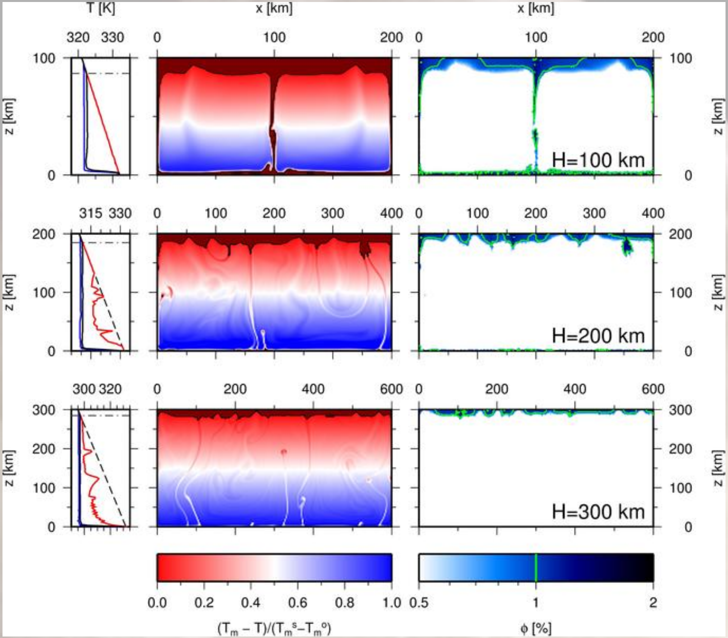


- silicates: T=T^m, ϕ~few %: volatiles leaching
- interior: T^av<T^m: melt can freeze; upwellings: volatiles transport
- ocean: T=T^m, ϕ~ϕ_c: volatiles dissolution into the ocean

Increasing HP ice layer thickness H ($\sim$ increasing Ra)



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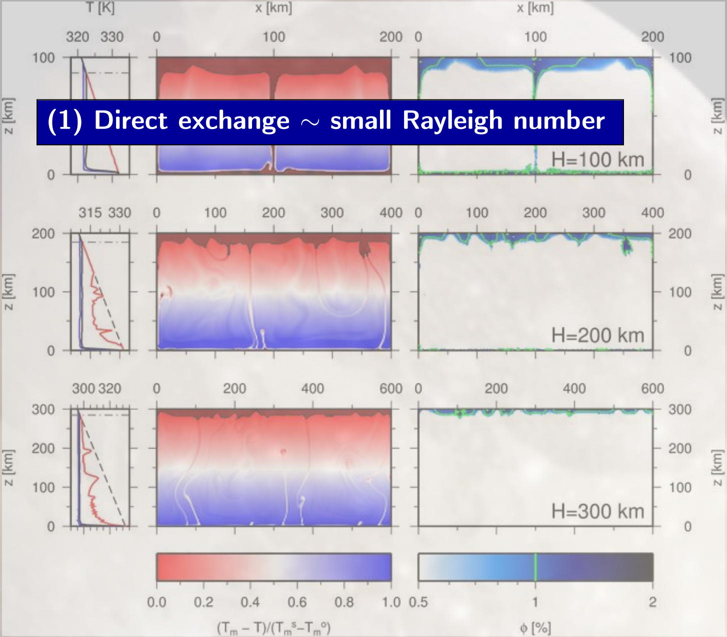

(1) Direct exchange $\sim$ small Rayleigh number

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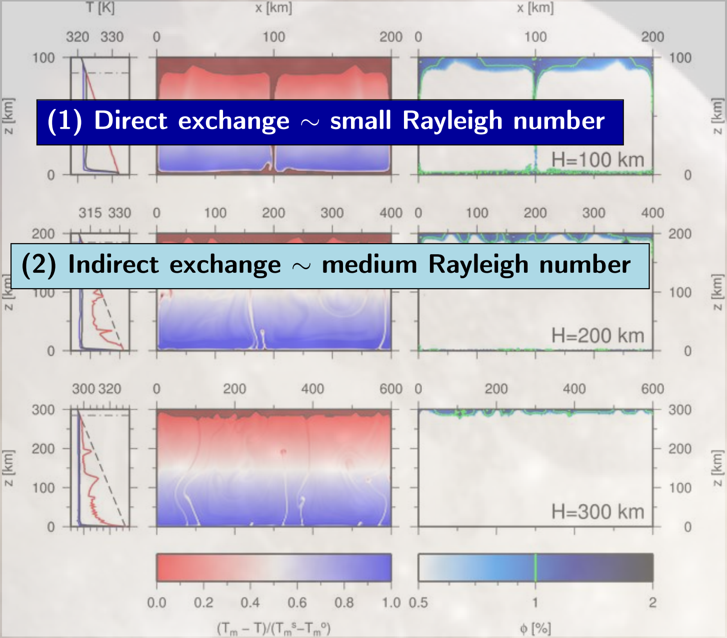

(1) Direct exchange $\sim$ small Rayleigh number

(2) Indirect exchange $\sim$ medium Rayleigh number

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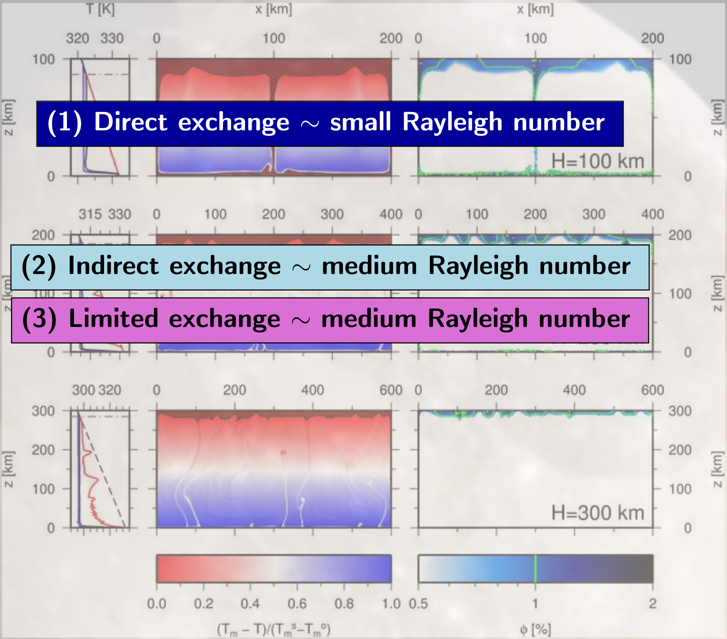

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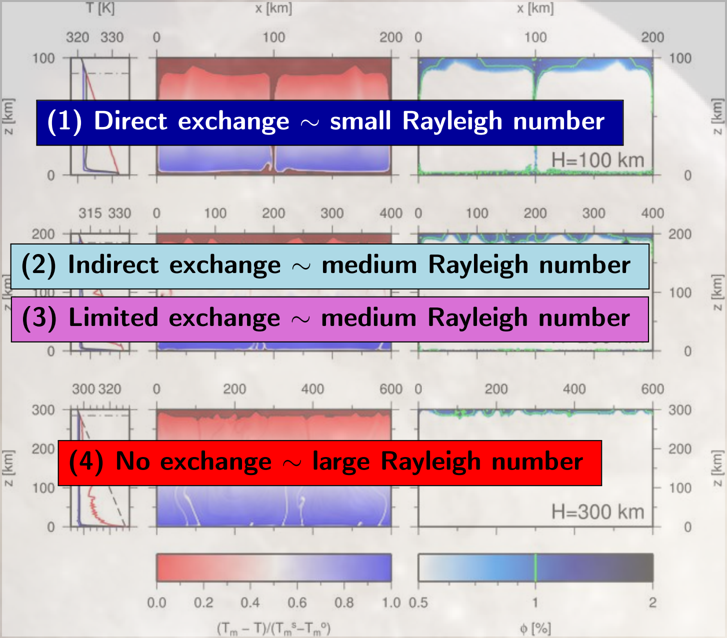
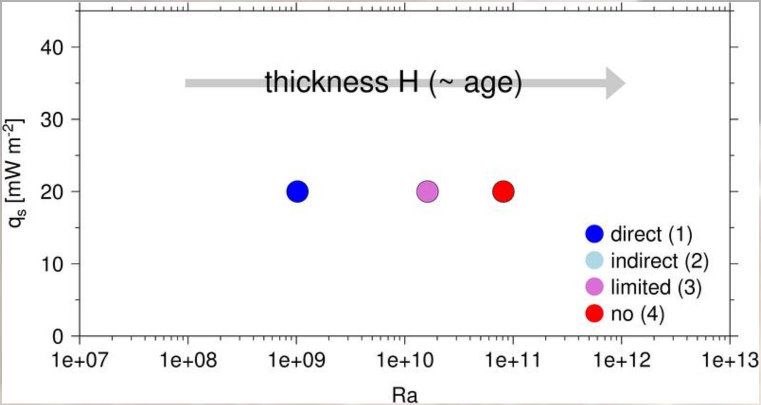


Results of parametric study



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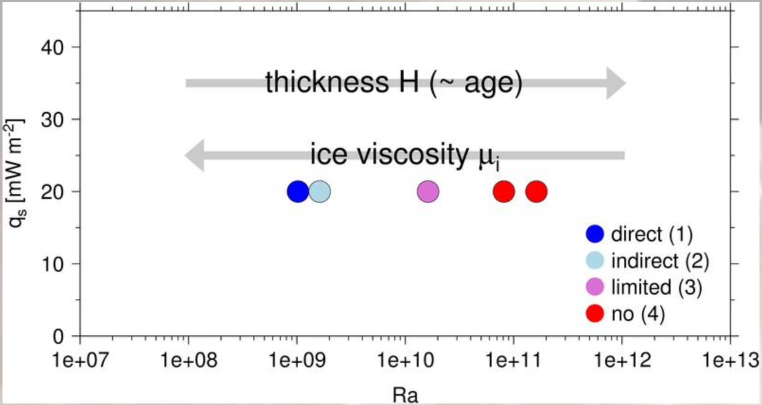

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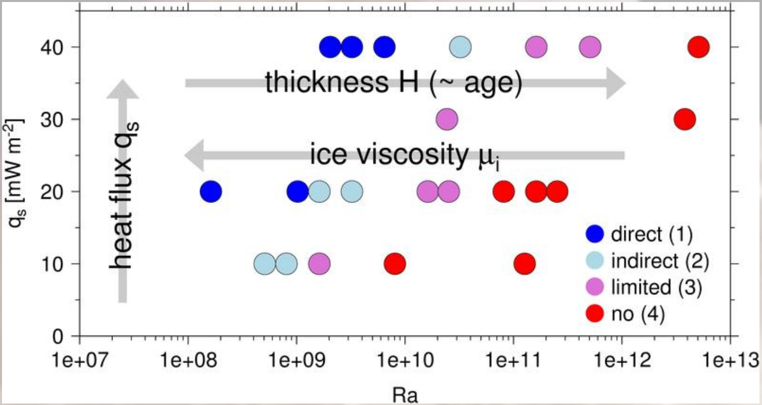

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Long-term evolution modeling

- 2d model: strong dependence on input parameters:
- HP ice layer thickness H
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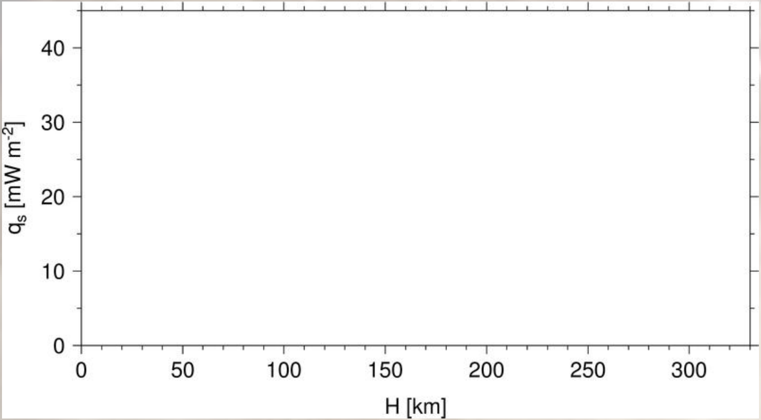
- 1d thermo(-chemical) evolution model:
- scaling laws for rocky interior, **HP ice layer**, ocean, ice I crust
- q_s and H are part of solution
- μ_i remains an input parameter

Critical heat flux for bottom melting

- fixed viscosity μ_i, range of q_s and H



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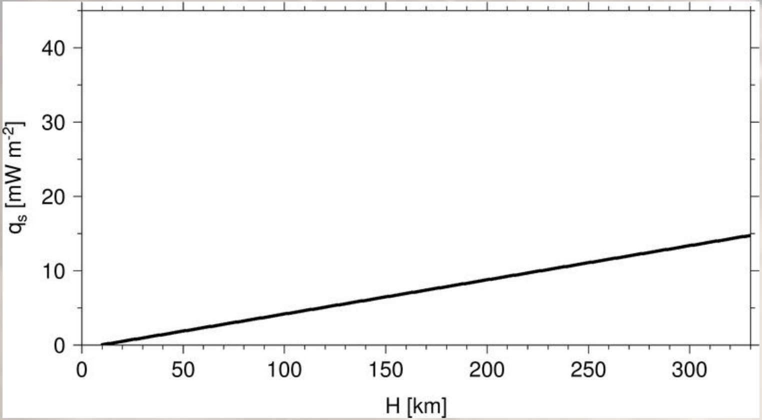
→ critical $q_{\rm s}^{\rm c}$ for bottom melting: $\boxed{q_{\rm s}^{\rm c} = \alpha\, H - \beta}$



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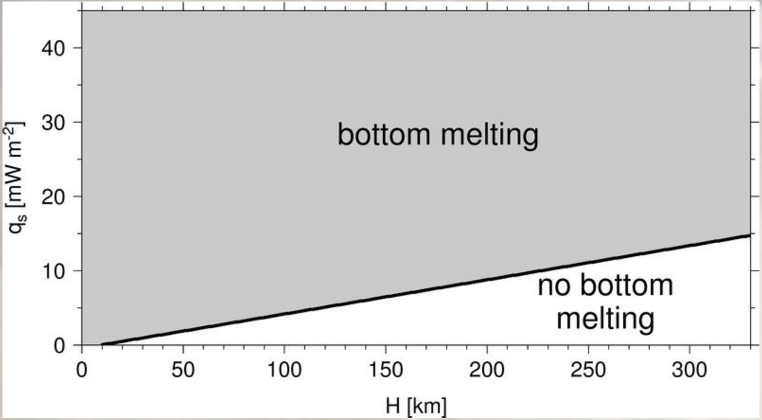

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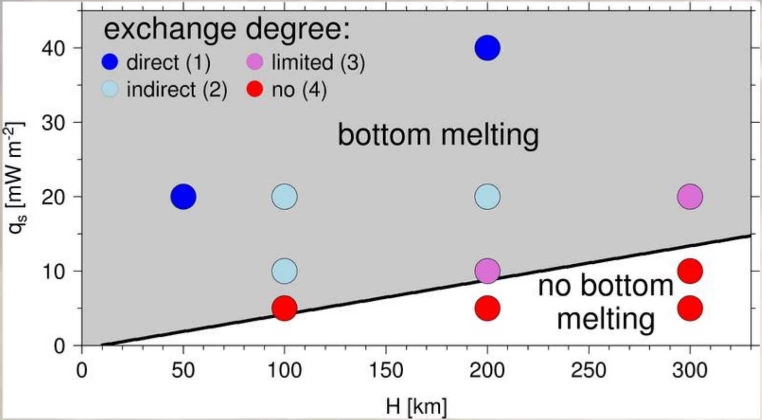
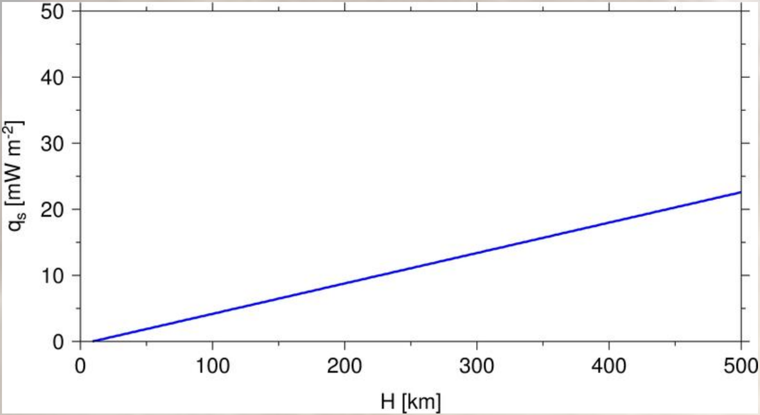


Present-day bottom melting: Ganymede vs Titan

- Titan, $\mu_i = 10^{15}$ Pa s: $q_s^c \propto H$



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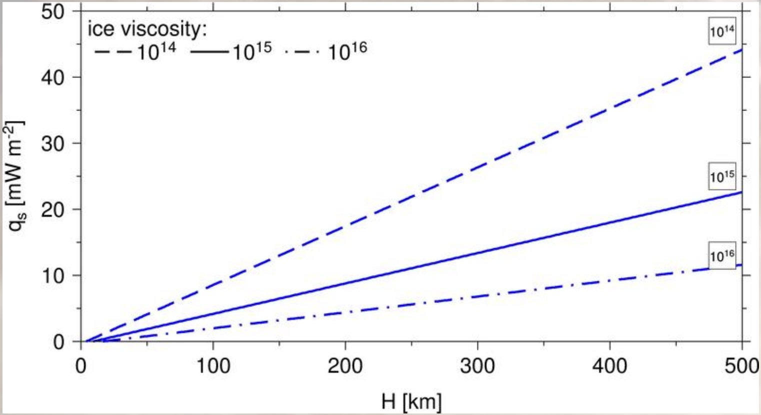

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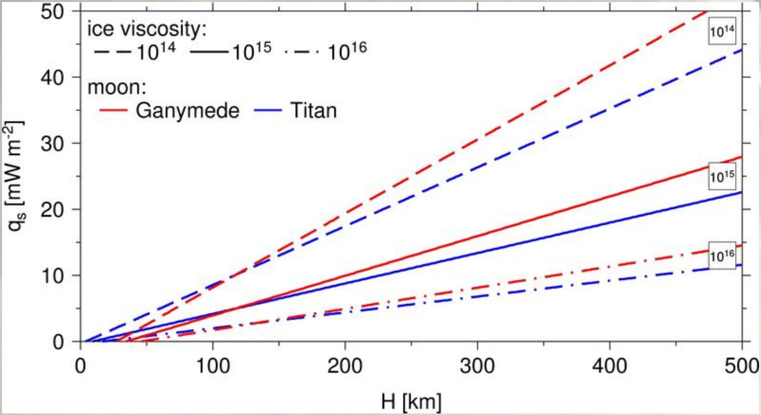

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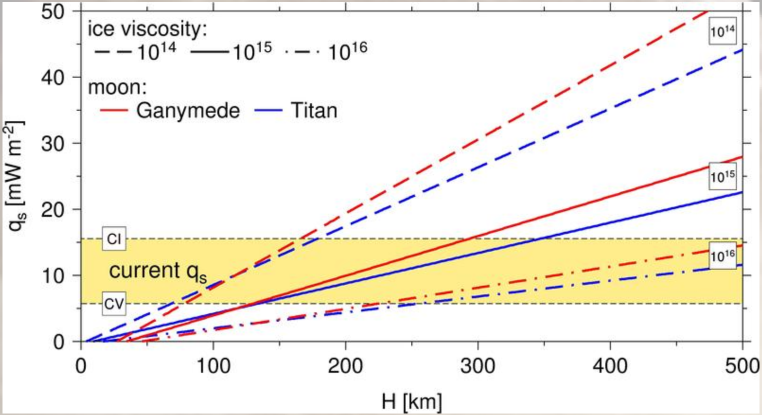

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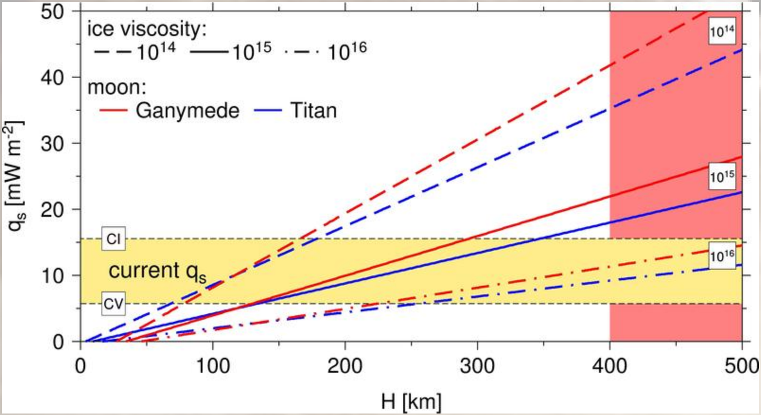

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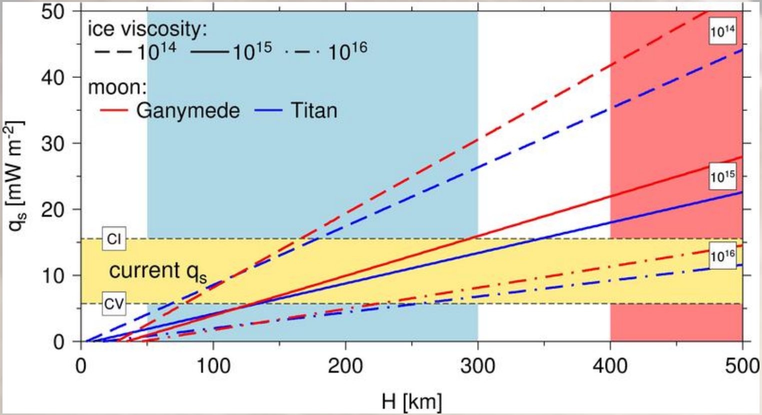

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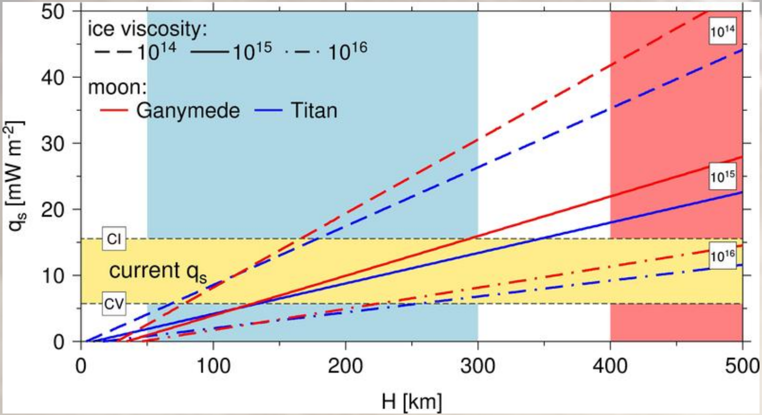

→ **Titan: melting at silicates interface and leaching of ^{40}Ar?**

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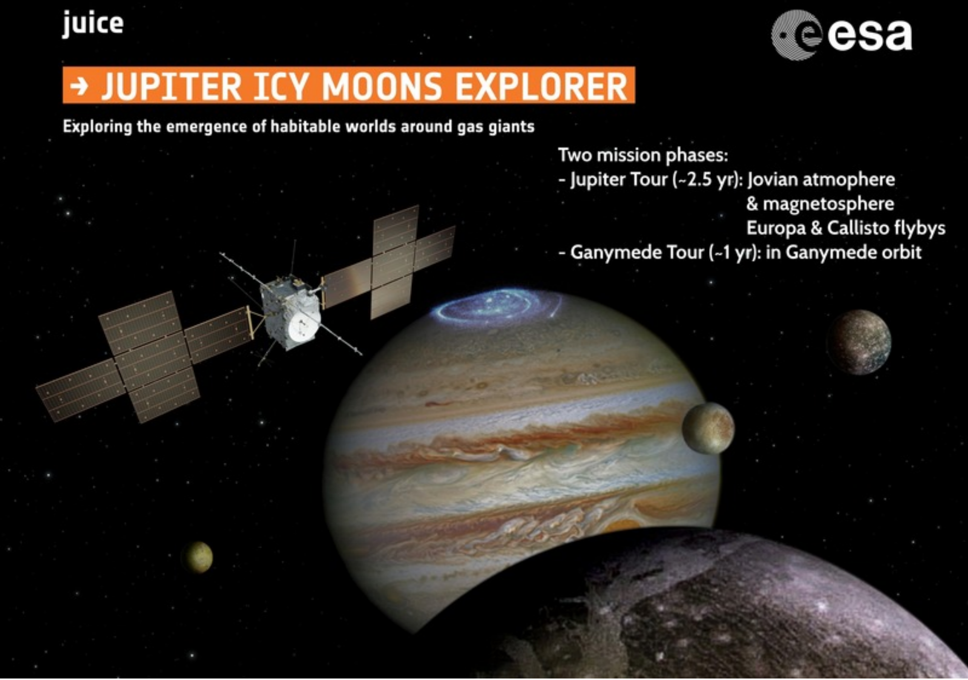
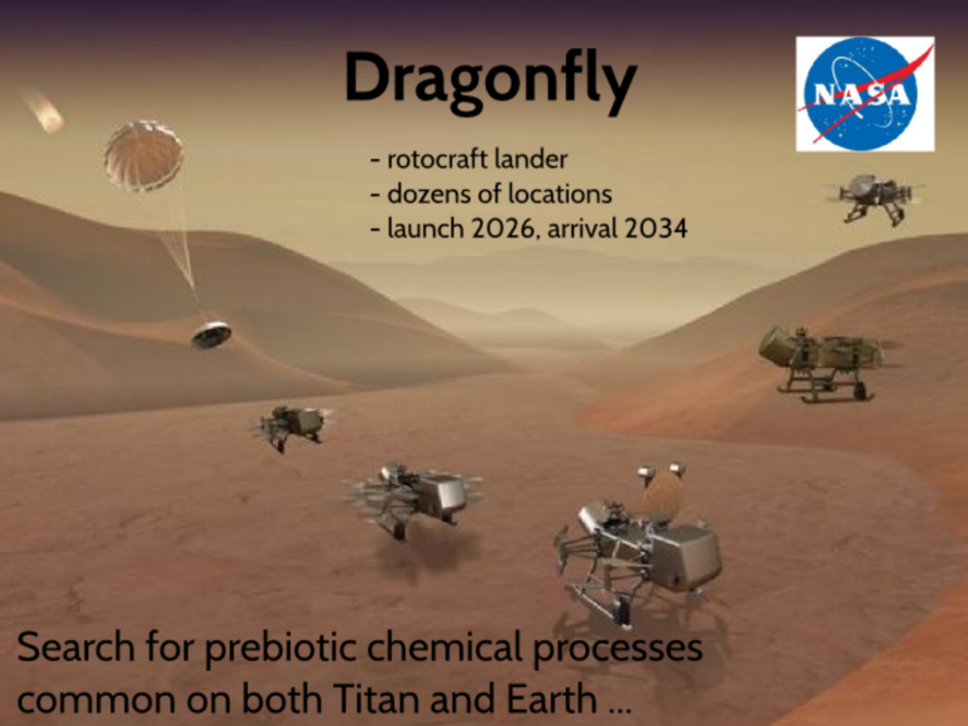

juice
esa
→ JUPITER ICY MOONS EXPLORER
Exploring the emergence of habitable worlds around gas giants
Two mission phases:
- Jupiter Tour (~2.5 yr): Jovian atmophere
& magnetosphere
Europa & Callisto flybys
- Ganymede Tour (~1 yr): in Ganymede orbit

Dragonfly

- rotocraft lander
- dozens of locations
- launch 2026, arrival 2034



Search for prebiotic chemical processes common on both Titan and Earth ...

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Thank you for your attention!

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