



## **An efficient hybrid optimization procedure of adaptive partition-based search and downhill simplex methods for calibrating water resources models**

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All models, including water resources models are a kind of simplified representations of the investigated “real-world” system. It is fairly typical that the model components and its attributes are aggregated into a limited number of physically meaningless parameters in order to facilitate the mathematical descriptions of the real-world processes. Consequently, these model parameters usually cannot be identified by measurements, therefore it is necessary to determine them via calibration applying optimization algorithms. As a rule, the optimality criterion is expressed by a nonlinear goal (loss) function, while the feasible domain for parameter selection is usually given by a closed interval of real numbers. Hence, the task of model-calibration frequently leads to solving a multiextremal global minimization problem: find an optimizer  $\underline{\mathbf{x}}^*$  such that generates a minimum of the real Lipschitz-continuous objective function  $f(\underline{\mathbf{x}})$ , which is defined on the finite interval  $P=[\underline{\mathbf{p}}_1;\underline{\mathbf{p}}_2]$  of the n-dimensional Euclidean space ( $\underline{\mathbf{x}}\in[\underline{\mathbf{p}}_1;\underline{\mathbf{p}}_2]=P\subset\mathbf{R}^n$ ). Although there are several classes of algorithmic global optimization (GO) approaches that have convergence properties, the development of a robust and really efficient GO software is still difficult, therefore a highly skilled optimization procedure has become a valuable tool in most of hydroinformatics applications.

In this presentation, a derivate-free hybrid method combining, globally convergent adaptive partition-based search (APS) and downhill simplex algorithm (DSA) is proposed to solve global optimization problems. The elaborated hybrid algorithm basi-

cally consists of two phases:

Firstly, the APS algorithm (Pintér – Szabó – Somlyódy) offers an adaptive global search (branch-and-bound) on the whole closed set of the feasible parameters. In these the parameter space  $\mathbf{P}$  is gradually divided into adaptively generated  $n$ -dimensional subintervals  $\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_m$ , using two specially constructed decision functions:

- a real-valued interval selection function  $R$  for choosing a subinterval from the already generated subintervals  $\{\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_m\}$  for partitioning, and
- a real-valued point selection function  $S$  for selecting an interior point of the subinterval selected by  $R$ , which new point defines partitioning of these subinterval to  $2^n$  new subintervals.

The goal function is evaluated in each “lower left” and “upper right” vertices of the  $2^n$  new subintervals. Denote by  $j$ ,  $m = m(j)$  and  $k = k(j)$  the current number of iteration of iteration cycle, generated subintervals and vertices of the subintervals  $\{\underline{\mathbf{x}}_1, \underline{\mathbf{x}}_2, \dots, \underline{\mathbf{x}}_k\}$ , respectively. It has been proved that if the decision (selection) functions  $R$  and  $S$  satisfy certain analytical requirements, then the set of points generated by the APS algorithm convergent to the solution of the problem:

$$\lim_{j \rightarrow \infty} \underline{\mathbf{x}}_{k(j)} = \underline{\mathbf{x}}^*.$$

Secondly, the incorporated DSA (Nelder – Mead) offers a local-search scheme starting from the “best” parameter vectors, which were detected in the previous phase by the APS. The simplex method is an efficient, robust iterative algorithm to solve unconstrained minimization problems numerically for several variables. A simplex is the geometrical figure in  $n$  dimensions consisting of  $n+1$  points (vertices) that span the  $n$ -dimensional space. After the definition of the initial simplex, the algorithm takes a series of iterative steps. The simplex moves, expands, contracts, and distorts its shape as it attempts to find a minimizer. In order to “keep” the algorithm in the  $n$ -dimensional interval  $[\underline{\mathbf{p}}_1; \underline{\mathbf{p}}_2]$ , it has been introduced a penalty function with a high value outside the specified constraints.

The combined (APS-DSA) algorithm does not require derivative information: the solver operations are based exclusively on the computation of goal function values at each algorithmically selected search point.

Following a brief description of the underlying theory, some comparative results are presented – based on a standardized test function suite from the computer science literature – in order to demonstrate the efficiency of the developed algorithm. Finally, the

methodology is illustrated by solving efficiently different type of real-world problems from the field of water resources management, such as:

- Calibrating some channel routing model.
- Calibrating distributed hydrological model.
- Determine optimal strategy of operational control for system of flood emergency reservoir.

**References:**

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Nelder, J. A. and Mead, R. *A Simplex Method for Function Minimization*. Comput. J. 7, 308-313, 1965.