



Minimum relative entropy update of ensemble probabilities to reflect forecast information

S.V. Weijs, N.C. van de Giesen

Water Resources department, Civil Engineering & Geosciences, Delft University of Technology [s.v.weijs@tudelft.nl]

In water resources management and hydrology, ensembles are widely used. They present a detailed stochastic description of phenomena, without the need to fit simplified models. Climatology can be represented by an ensemble of past measured time series. When long term forecasts are available, climatic uncertainty can be reduced with actual information, resulting in a probabilistic forecast. Often, long term forecasts do not contain information at a high spatial and temporal resolution, but rather contain information about averages in time and space. A reasonable way to combine the detailed information in the historical ensemble of time series with forecast information is to update the ensemble probabilities, while leaving the ensemble members intact. The question now is how to update these probabilities.

We present a method that exactly uses all forecast information in the least-biased way. The method is an optimization of the updated probabilities, minimizing relative entropy (also known as Kullback-Leibler divergence). The well founded information-theoretic concept of minimum relative entropy (MRE) ensures the least-biased estimate for the distribution, given the literal interpretation of the forecast information, provided as constraints. The MRE is an exact measure of the amount of information in the forecast. The expectation of MRE over all possible forecasts is equal to the mutual information between the forecast and forecasted quantities. The MRE-update method updates the a priori equal probabilities of the scenarios to reflect the forecast information, based on the averages in space and time derived from the time series, compared to information on these same quantities in the forecast.

Previous research has addressed the same updating problem for the case in which forecast information is given as a conditional mean and variance or in the form of tercile probabilities. One method adjusts the ensemble probabilities by the ratio between the marginal and assumed conditional probability density functions (pdf-ratio). Another method formulates an optimization problem similar to ours, imposing the new mean and variance or tercile probabilities as constraints. The difference is in the objective function, which uses minimum squared adjustment of probabilities.

In contrast to these previous methods, the objective in the MRE-update method provides an update that is free of assumptions, except for the literal interpretation of the forecast information provided. In this way, the method also shows the difference between forecast information in the form of conditional mean and standard deviation and in the form of updated tercile probabilities. In the first case, results are close to those of the pdf-ratio method with assumed normal distributions, except for cases when extraordinary events are forecasted. The MRE-update then assigns a relatively higher probability to the more extreme ensemble members. In the second case, the MRE-update results in a block adjustment, assigning equal probabilities to all ensemble members within one tercile. This leads to the conclusion that updated tercile probabilities are not an adequate way to represent a smooth forecast distribution.