Geophysical Research Abstracts,
Vol. 10, EGU2008-A-00900, 2008
SRef-ID: 1607-7962/gra/EGU2008-A-00900
EGU General Assembly 2008

From symmetry break to Poisson point process in 2D Voronoi tessellations: the generic nature of hexagons

V. Lucarini (1,2)

(1) Department of Physics, University of Bologna, Bologna, Italy [lucarini@adgb.df.unibo.it]
(2) CINFAI, Camerino, Italy

We bridge the properties of the regular triangular, square and hexagonal honeycomb Voronoi tessellations of the plane to those of the Poisson-Voronoi case, thus analyzing in a common framework symmetry-break processes and the approach to uniformly random distributions of tessellation-generating points. We resort to ensemble simulations of tessellations generated by points whose regular positions are perturbed through a Gaussian noise, whose variance is given by the parameter alphaˆ2 times the square of the inverse of the average density of points. We analyze the number of sides, the area, and the perimeter of the Voronoi cells. For all values alpha>0, hexagons constitute the most common class of cells, and 2-parameter gamma distributions provide an efficient description of the statistical properties of the analyzed geometrical characteristics. The symmetry break induced by the introduction of noise destroys the triangular and square tessellations, which are structurally unstable, as their topological properties are discontinuous in alpha=0. On the contrary, the honeycomb hexagonal tessellation is topologically very stable and, experimentally, all Voronoi cells are hexagonal for small but finite noise with alpha<0.12. A notable signature of the symmetry break occurring for all tessellations is the observed linear dependence of the ensemble mean of the standard deviation of the area and perimeter of the cells on alpha; for small values of alpha. Already for a moderate amount of Gaussian noise (alpha>0.5), memory of the specific initial unperturbed state is lost, because the statistical properties of the three perturbed regular tessellations are indistinguishable. When alpha>2, results converge to those of Poisson-Voronoi tessellations. The geometrical properties of n-sided cells change with alpha until the Poisson-Voronoi limit is reached

for alpha>2; in this limit the Desch law for perimeters is confirmed to be not valid and a square root dependence on n is established. This law allows for an easy link to the Lewis law for areas and agrees with exact asymptotic results. Finally, for alpha>1, the ensemble mean of the cells area and perimeter restricted to the hexagonal cells agree remarkably well with the full ensemble mean; this reinforces the idea that hexagons, beyond their ubiquitous numerical prominence, can be interpreted as typical polygons in 2D Voronoi tessellations.