



Formulating an inverse problem to determine the accumulation rate pattern from deep internal layering in an ice sheet

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Formal inverse methods to estimate the value of model parameters based on some measured values of data parameters have been used more widely in glaciology in recent years. We determine the accumulation-rate profile along a flow band defined by the surface topography, the influx of ice into the upstream end of the flow band, and the age of an internal layer. The data comprise the depth profile of the internal layer, a few velocity measurements at the surface, and the average accumulation at one location. The data in our example were collected at Taylor Mouth, a flank site of Taylor Dome, Antarctica.

Formulations of inverse problems can be divided into three categories: 'Smallest', 'Flattest', and 'Smoothest'. In a 'Smallest' formulation, the *a priori* probability distribution is defined by the distance between the model parameters and their respective preconception values. Other formulations are possible when model parameters have a geometrical relationship. In a 'Flattest' formulation, the *a priori* probability distribution is defined by the deviation of the slope of the model-parameter profile from a slope of zero. In a 'Smoothest' formulation, the *a priori* probability distribution is defined by the deviation of the curvature of the model-parameter profile from zero curvature.

Solutions to inverse problems are non-unique in the sense that no single set of model parameters can be selected with certainty; the solution can be expressed only through a probability density in the model-parameter space. Furthermore, the probability density can have multiple maxima implying that the solution to the inverse problem is

multimodal. Therefore using as many different formulations and tools as possible can reveal more information than any single technique alone. This Taylor Mouth problem was solved previously using a gradient method. A gradient method finds only one maximum. To compare with the gradient-method solution, and to check for multiple maxima in the probability density, we solved this inverse problem with a Monte Carlo simulation using the Metropolis algorithm, which systematically searches the model-parameter space for maxima in the probability density.

Based on our *a priori* information, and on desirable constraints on the accumulation-rate profile, we solve the Taylor Mouth inverse problem first using a mixed ‘Smallest’ and ‘Smoothest’ formulation, and then using a pure ‘Smallest’ formulation. We find that this particular inverse problem can have up to four different maxima in the probability density, each corresponding to a different accumulation-rate profile. These different accumulation-rate profiles resemble the solution from the gradient method. Since no smoothness constraint is imposed on the pure ‘smallest’ formulation, its solutions for the accumulation-rate profile are rougher, but the general trends are similar. The different solutions infer different ages for the observed layer. Older layer-age estimates are paired with slower ice flow, which is achieved through reduced accumulation rate, and reduced influx of ice into the upstream end of the flow band. We find that the solution from the gradient method corresponds to the most likely solution from the Monte Carlo simulation. Comparison of the ‘Smallest’ and the ‘Smoothest’ solutions also revealed the existence of temporal change in the accumulation rate.