



Absolutely stable explicit scheme for the diffusion equation

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The paper presents a way to construct an explicit scheme for the diffusion equation that has no limitations concerning stability. It was constructed on the basis of a representation of integral solution of approximated linear issue. Results of investigation of this scheme's properties were compared with other commonly used differential schemes for the diffusion equation. This scheme provides high accuracy solutions of the diffusion equation at high numerical effectiveness. The solution of the 1D-space diffusion problem was considered with the assumption that it would be possible to enhance it on more dimensions by means of the decomposition method. In the beginning, the problem with a constant diffusion coefficient was discussed and then enhanced with discussion concerning a variable diffusion coefficient. The non-dimensional form of equation: $u_t = u_{xx} + f(x, t)$ was used with the following initial condition: $u(x, 0) = u^0(x)$. It has an accurate solution in an integral form which was used to construct the method. The approximate solution was constructed on the basis of assessing of the specific elements of the integral solution. The sums of infinite number of elements in the solution were replaced with sums of finite number of elements after an appropriate renormalization of the weighting coefficients. Applying averaged diffusion coefficient (instead of its current value) enables application of the presented method to solving diffusion issues for coefficients varying in space in a regular way. We then propose to apply analytical enhancement of the diffusion coefficient beyond the area of consideration to the distance that enables application of integral solution (as for constant diffusion coefficient). Numerous numerical tests are presented for various forms of the boundary conditions. The obtained results suggest that the method has both advantages and disadvantages. It is its undoubted advantage that the method

is explicit and unconditionally stable (what has not been noticed so far in numerical practice for the diffusion issue). Another advantage is that it is an effective method as far as time of computing is concerned which may be crucial for operational solving of the issue of pollutants and contamination spreading in the atmosphere. Unquestionable disadvantage is that the method requires careful and time-consuming preparation for numerous specific issues. These are the algorithms of analytical enhancement of the functions beyond the area taken into account. It concerns the issues of enhancing the initial condition, diffusion coefficients, intermediate results obtained at the individual time layers, etc. Executing all the actions (preparing adequately effective algorithms), e.g. for an operationally active model of atmospheric air contamination propagation is finally profitable. It makes the computing time shorter which may be the most important issue.